Fear of Secular Stagnation and The Natural Interest Rate

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MOTIVATION

How can we explain the sudden drop in interest rates in the aftermath of the financial crisis?

Plausible causes:

- A decrease in productivity that occurred during the crisis
- A change in the **agents' beliefs**

Goal: Study the role of agent's beliefs and pessimism in explaining the drop in interest rates during the Great Recession

Assumption: Uncertainty about the nature of the shocks that hit the economy: was the decline in GDP persistent but temporary, or permanent?

Conjecture: The attribution of a positive probability to the scenario of secular stagnation acts "per se" as a force that induces a more cautious behavior

This paper:

- Verify if this conjecture is empirically relevant
- Quantify the role of beliefs and pessimism in explaining the decline of the interest rates

METHODOLOGY

THE ENVIRONMENT

General equilibrium model with growth, where

- The agents do not observe the determinants of productivity
- They take into account this uncertainty in their decision making process
- They can be pessimist
- Uncertainty over the components of productivity and pessimism can vary over time

THE PROCESS FOR TECHNOLOGY

Technology is described by the following DLM:

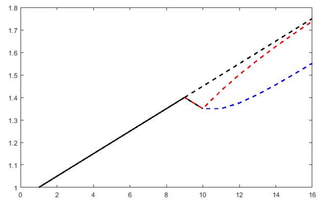
$\ln(A_t) = l_t + f_t$

$l_t = l_{t-1} + \gamma_t$

$\gamma_t = (1-\rho_\gamma)\,\bar{\gamma} + \rho_\gamma \gamma_{t-1} + \sigma_\gamma \epsilon_{\gamma,t}$

$f_t = \rho_f f_{t-1} + \sigma_f \epsilon_{f,t}$

$(\epsilon_{f,t}, \epsilon_{\gamma,t})' \sim N(0, I)$



The agents observe A_t, but not its components $\theta_t = [\gamma_t \; f_t \; l_t]'$, and do not observe the realization of $\epsilon_{\gamma,t}$ and $\epsilon_{f,t}$. Parameters are known. The agents face *Ambiguity*

A simple example: endowment economy

THE PREFERENCES

Recursive smooth ambiguity preferences (Klibanoff, Marinacci, Mukerji, 2009)

$$V_{s^t}(B_t, \mu_t) = \max_{C_t, B_{t+1}} \ln(C_t) + \beta\phi^{-1}\left[E_{\mu_t}\phi\left(E_{\theta_t} V_{(s^t, A_{t+1})}(B_{t+1}, \mu_{t+1})\right)\right]$$

- *Ambiguity*: characterized by the variance of the posterior distribution μ_t.
- *Ambiguity attitude*: characterized by the shape of ϕ
  - concave: ambiguity averse (pessimist)
  - linear: ambiguity neutral (Bayesian)
  - convex: ambiguity loving (optimist)

Assume $\phi(y, \alpha_t) = -\frac{1}{\alpha_t}\exp\{-\alpha_t y\}$

α_t: (time varying) coefficient of ambiguity attitude

THE EQUILIBRIUM CONDITIONS

$$1 = E_{\mu_t}\left[\xi_t(\theta_t)\, E_{\theta_t}\left(\beta \frac{A_t}{A_{t+1}}\right)\right] R_{t+1}$$

$$\ln\left(\frac{A_{t+1}}{A_t}\right) = (1-\rho_\gamma)\,\bar{\gamma} + \rho_\gamma \gamma_t + (\rho_f - 1)\, f_t + \sigma_\gamma \epsilon_{\gamma,t+1} + \sigma_f \epsilon_{f,t+1}$$

where :

$$\xi_t(\theta_t) \equiv \frac{\exp\{-\alpha_t E_{\theta_t} V_{t+1}\}}{E_{\mu_t}\left[\exp\{-\alpha_t E_{\theta_t} V_{t+1}\}\right]}$$

- ξ_t is a Radon-Nikodym derivative with respect to the Bayesian posterior distribution: $d\mu_t^* = \xi_t d\mu_t$
- ξ_t creates a wedge between the expectations of a bayesian agent and of an ambiguity-averse agent: *pessimism*

THE BELIEFS DISTORTION

Time variation in the two sources of pessimism:

- *Ambiguity attitude*: α_t
  We assume, as in Bhandari, Borovicka and Ho (2019):
  $\alpha_t = (1-\rho_\alpha)\bar{\alpha} + \rho_\alpha \alpha_{t-1} + \sigma_\alpha \epsilon_{\alpha t}$
- *Ambiguity* : the variance of the posterior distribution
  - Under μ_{t-1}, $(\theta_{t-1}|A_{t-1}) \sim N(m_{t-1}, Q_{t-1})$
  - In standard filtering problem this posterior distribution becomes the prior to update beliefs over θ_t
  - We assume time variation in uncertainty through a shock to the variance of the prior distribution:
    $Q^*_{t-1} = Q_{t-1} e^{\sigma_\eta \eta_t}, \;\; \eta_t \sim N(0,1)$
  - Without the shock η_t, Q_t converges to time invariant variance of the steady state Kalman filter

THE APPROXIMATED SOLUTION

- Joint perturbation of variance of the shocks and coefficient of ambiguity aversion (Borovicka and Hansen, 2014)
- We apply this idea to models with smooth ambiguity preferences: additional challenge to keep track of the evolution of beliefs

The approximated beliefs distortion:

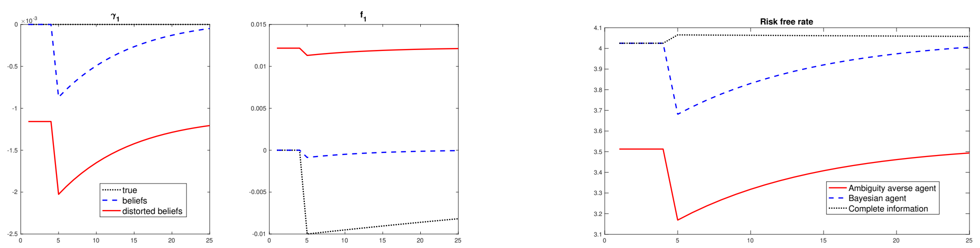
- Under the posterior distribution μ_t: $\theta_t \sim N(m_t, Q_t)$
- Under the distorted distribution μ^*: $\theta_t \sim N(m_t - \alpha_t Q_t B', Q_t)$
  - *Ambiguity aversion* affects only the mean
  - *Ambiguity* affects both the mean and the variance

Approximated solution of the simple model:

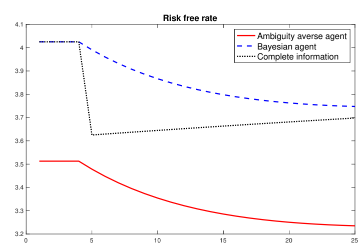
$$R_{1t} = \beta^{-1} e^{\bar{\gamma}} \begin{bmatrix} \rho_\gamma & \rho_f - 1 & 0 \end{bmatrix} \Bigg[m_{1t} - \underbrace{(\bar{\alpha} Q_{1t} + Q\alpha_{1t} + \bar{\alpha} Q)\, B'}_{Pessimism} \Bigg]$$

IMPULSE RESPONSE FUNCTIONS

EFFECT OF A NEGATIVE TEMPORARY SHOCK



EFFECT OF A NEGATIVE PERMANENT SHOCK



THE WORK AHEAD:

- The core mechanism in a more realistic model
- Estimate the model to quantify the role of beliefs and pessimism in explaining the drop in interest rates
- Disentangle the sources of pessimism

[1] The views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.