Some historical and methodological remarks on Giuseppe Palomba's topological dynamics

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Abstract

Giuseppe Palomba, second-generation member of the Italian Paretian School, devoted his activity to devise an original axiomatic framework for economic dynamics, primarily designed to address economic phenomenal changes. The framework consists in a sequence of axiomatic layers (each one representing an economic phenomenon), where the subsequent layer encompasses the precedent as special case. This essay proposes the interpretation of such framework through the methodological notion of 'structural realism', whose importance for economics has been emphasized by Ross (2008). The objective of this essay is that of assessing the intellectual contribution of Palomba, identifying him as a proto-structural realist in economics.

Keywords: Italian Paretian School; Pan-marginalism; Invariances; Structuralism; Realism.

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1. Introduction

In the history of mathematical economics there are still undiscovered treasures: this essay claims that Giuseppe Palomba is one of them. Second-generation member of the Italian post-Paretian School, pupil of Luigi Amoroso, Palomba spent his whole intellectual life trying to draw a sound connection between economics and contemporary (i.e. post-Newtonian) physics, reaching highly distinctive results. It is because of the original conceptual apparatus he devised for economic dynamics, and because of the sophisticated mathematical tools of group theory he employed, that Palomba's economics received little attention from his contemporaries. Such attitude has propagated in current research in the historiography of economic thought, so preventing economists of today from getting in touch with his view on economics. It is the primary objective of this essay that of providing an account of Palomba's economic thought, in order to assess his distinctive and pioneering contribution.

This essay, further, tries to pursue a second and wider objective: that of giving a specific interpretation of Palomba's economic framework, reconstructing it in light of the interpretative category of 'structural realism', as emerged in the philosophy of science literature in the last few decades. Palomba's conceptual schema expresses – this essay argues – the basic features of a 'structural realist' point of view on economic theories, so that Palomba is identified in what follows as a proto-structural realist thinker in the field of economics. Research in the history of economic thought is so intended to interact constitutively with the too often separated research in philosophy and methodology of economics. This essay, endorsing Don Ross' judgment (2008) that structural realism is a category particularly important for the reconstruction of mathematical economics theorizing, is meant to open an historical front in which to assess such judgment.

The essay is organized as follows. Section 2 provides a necessary, although brief, introduction to the notion of 'structural realism' as general interpretative category. Section 3 introduces the framework set up by Palomba to study economic dynamics and change. Section 4, lastly, identifies the structural realist features of Palomba's thought, allowing us to draw some broad remarks concerning the hypothesis of structural realism in economics.

2. What is it like to be a 'structural realist'?

This section introduces briefly the notion of 'structural realism' as it has been conceived in the philosophy of science literature. This is a necessary step in order to prove, in the next Sections, the consistency of our interpretation of Palomba's contribution to economics as a piece of structural realism. This Section can however be read, autonomously, as an introduction to the notion of structural realism for the philosophically interested historian of economic thought, since such notion concerns the issue of discontinuity and change in the history of theories, a primary interest for each historian of analytical thought.

Let me start with a couple of preliminary clarifications. First, it is helpful to note that the notion of 'realism' in the philosophy of science literature has a meaning distinct from the way the term is used in economics; there "is a major terminological discontinuity between the two disciplines" (Mäki 2008: 334), which must not be overlooked. Usually, economics does not distinguish between 'realism' and 'realisticness'[1], and uses the former in the sense of the latter (Mäki 1989). In this essay, the notion of realism is used according to the meaning aroused in the philosophy literature. Second, the notion of 'structural realism' can most easily be understood as a particular form of the broad field of inquiry named 'scientific realism'. 'Scientific realism', roughly speaking, is centrally concerned with the relationship that science – and scientific theories in particular – bears with 'truthfulness'. In this respect, the ultimate goal of 'scientific realism' is to identify the conditions under which scientific theories can be considered as having 'truth-value'[2]. On the other hand,

[1] Maki (1989: 194) distinguishes between "realism as designating a collection of ontological and semantic doctrines" from "realisticness as designating a collection of attributes predicable of representations".
[2] For an overview of the notion of 'scientific realism' see Chakravartty (2011) and references therein.

‘scientific anti-realism’ denies that scientific theories can be considered as true or false, claiming instead that they should be assessed on the basis either i) of their instrumental value or ii) of their empirical adequateness (see Van Fraassen 1980).

Within the field of scientific realism, one important position is ‘structural realism’, which has been recently put forward by the LSE philosopher John Worrall[3]. Worrall describes structural realism as a position reconciling scientific realism and scientific anti-realism, under the attractive perspective that their theoretical disputes would be synthesized by taking “the best of both worlds” (Worrall 1989). In particular, structural realism would reconcile and do justice to the strongest arguments of both realists and anti-realists: respectively, the ‘no miracles’ argument (e.g see Musgrave 1988) and the ‘pessimistic meta-induction’ argument (see Laudan 1981). Realists claim that it would be a ‘miracle’ if scientific theories were at the same time effective (as they are, mainly in terms of predictions) and not true (in terms of not being able to say anything about the substantive reality). On the other hand, anti-realists claim that talking of the ‘approximation’ of theories to truth would require some form of continuity among theories in time, in the sense that changes across theories should be in some way cumulative. Clearly, this is not always the case: think of the status of ‘incommensurable’ scientific theories as described by Kuhn (1962). The aim of structural realism is to overcome this incompatibility of views by considering the *structural* dimension of theories. In particular, structural realism emphasizes continuity and/or permanence (in other words, some form of invariance) of *mathematical structures* among theories. By means of this characterization, i) the ‘no miracles’ argument would be met simply by attributing truth value primarily to structures and ii) the ‘pessimistic meta-induction’ argument would be met by considering structures as invariants with respect to the incommensurable (non-mathematical) content of theories. The often-cited historical example brought up by structural realists in support of their view is that of the classical wave theory of light (based on the assumption of existence of elastic solid ether) of Augustin-Jean

[3] Worrall indicates as precursors the mathematician Henri Poincaré and the philosopher Glover Maxwell. See references in Worrall (1989).

Fresnel, superseded by the electromagnetic theory of light of James Clerk Maxwell. The latter denies the existence of ether at all. Nonetheless:

> [t]here was continuity or accumulation in the shift, but the continuity is one of *form* or structure, not of content […] if we restrict ourselves to the level of mathematical equations – *not* notice the phenomenal level – there is in fact complete continuity between Fresnel's and Maxwell's theories" (Worrall 1989: 117; 119, italics in the original).

This example goes hand in hand with that of mathematical continuity between Newtonian and Einsteinian physics; they constitute some of the most strikingly successful historical examples of structural continuity, supportive of the hypothesis of structural realism. To sum up, entities whose truth value is asserted are mathematical structures[4].

Going a step deeper into structural realism, James Ladyman (1998) distinguishes, as a refinement of Worrall's position, between an Epistemic Structural Realism (ESR) and an Ontic Structural Realism (OSR). Stathis Psillos puts the distinction in the following terms[5]:

> SR has two options available. Either there is something other than the structure – call it X – in the world, which however cannot be known, or there is nothing else in the world to be known. On the first disjunct, the restriction imposed by SR is epistemic. […] On the second disjunct, the restriction is ontic: there is nothing other than the structure to be known, because there is nothing other than structure (2001: S18-S19)

The gist of the distinction is in the way to conceive the relationship between (mathematical) *structures* and their so-called *objects of predication*, i.e. the semantic content of these structures. For ESR, objects of predication are something unknowable, in a Kantian fashion (Ladyman, 2009), leaving space and prominence to the knowledge of structures alone. Instead, according to OSR, "there are no unknowable *objects* lurking in the shadow" (French 1998: 203, italics in the original). While OSR denies "that there are objects that are not structures" it nonetheless claims that "there will always be a place for objects, understood as objects of predication; but [there is] no reason why objects in this sense should precisely line up with the constituents of reality" (Saunders 2003: 130).

[4] Beyond a simplistic notion of mathematical structure, Chakravartty (2004) claims that mathematical structures relevant to structural realism are structures that express "relations between first order, causal properties" in the languages of logic or mathematics.

[5] For further refinements of this distinction see Ladyman and Ross et al. (2007) and Ladyman (2009).

To take into account all the relevant positions, on a skeptic side, Psillos maintains the impossibility of separating, both epistemically and ontologically, the content and the structure of entities that, as he claims, "form a continuum" (Psillos 1995: 15).

As far as we are concerned in this essay, a disciplinary remark is key: structural realism has been originally developed as a realist position for physics. It is a recent turn in the research on structural realism to consider its extension to other scientific domains (Ladyman 2008), among which are social sciences (Kinkaid, 2008). Ross (2008), supporting an OSR point of view on economics, claims that "one […] should not expect to motivate OSR by appeal to disciplines such as sociology that lack distinctive canonical formal theory for interpreting quantitatively parameterized models", and that instead "among social sciences, economics has the requisite mathematical structure, in which it expresses a suite of standard theories" (ibid.: 733). According to Ross, the OSR framework, as it has been devised for physics, can be projected into economics. The common OSR framework for both physics and economics would be guaranteed by the homogeneous "metaphysical significance" (ibid.: 741) of objects of predications within the two domains of inquiry. There would be "structures" and their relative objects of predications ("relata") in the domains of both physics and economics: just in the same way rocks and tables are the objects of the structure of the force of Newtonian gravity, people are the objects of the structure, for instance, of economic games. "In this respect, economic theory exactly resembles physical theories, just as Jevons and Walras hoped it would" (ibid.: 742), realizing a long-lasting dream in economic theory (see Mirowski 1989). In such framework, it is important to emphasize that the justification of structural realism as the (effective) application of the same formal theories (as mathematical structures) in different domains of inquiry (e.g. physics and economics) is supported by the specific view of theories as 'semantic' or 'model-theoretic' constructs[6] (see e.g. Chakravartty 2001).

[6] The 'semantic' or 'model-theoretic' view of theories sees theories as 'families of models', where the word model has to be intended in the sense of formal systems. In this acception, models are the 'interpretations' of the formal, mathematical structure. This is the reason why the model-theoretic view of theories is consistent with the cross-

A huge debate has developed in philosophy of economics, precisely about the possibility of borrowing 'realist' frameworks originally devised for physics. The outcome of that debate consisted in recognizing either the "irrelevancy" of the notion of realism in economics if borrowed from physics (Hausman 1998) or the necessity to adjust the realist framework of physics to the specificities of economics (Mäki 2000). In brief, the point to be ascertained is whether 'structural realism' has to be taken either as a *monist* or a *pluralistic* position, i.e. whether there is a notion of structural realism that is good for all disciplines or not (for monist vs. pluralistic realisms see Mäki 2011).

A last point, of a rather technical nature – which has however much to do with the scope of this essay – is related to the importance of the mathematics of *group theory* for the individuation and characterization of structures. A huge number of scholars, who would define themselves as 'structural realists', believe that the identification of structures is above all a matter of identifying the right groups of transformations in which to express phenomena and theories. Not surprisingly, the privileged field of application of group theory has been physics and, in particular, quantum physics (French 1998) and relativity theory. It is not by chance that people like Hermann Weyl and Arthur Eddington, who championed the use of group theory in physics, were among the most influential precursors of structural realism (Ladyman 2009). In this sense, group theory has historically constituted the most important technical apparatus conducive to the structural realist hypothesis.

I can sum up by isolating, from the above considerations, a set of propositions that could characterize a 'minimalist structural realist' position[7]:

- structures are conceived to have truth value, either epistemically (ESR) or ontologically (OSR);

domain conception of theories: the same mathematical structure (i.e. the same formal theory) can be interpreted as holding in different domains (e.g. both in physics and economics).

[7] Similar 'minimalist' attempts have been made not infrequently, see e.g. Kincaid (2008).

- these structures are mathematical structures;
- mathematical structures are identified mainly by means of groups of transformations;
- groups of transformations are able to allow the identification of invariant structures across radically different phenomena and theories, i.e. across paradigm changes.

3. Giuseppe Palomba's view of economic dynamics

The figure of the economist Giuseppe Palomba is almost completely forgotten[8]. Typically, dismissive accounts of Palomba's work emphasize how his ultimate framework is distinctively *sui generis*. This judgment has been repeated so often that one is led to wonder whether the thin line between heterodoxy and idiosyncrasy is for Palomba irremediably crossed, compromising the possibility of reaching a meaningful and effective reconstruction of his thought. It is the aim of what follows that of showing that behind such *sui generis* thought there is something that is not only perfectly consistent, but also worth to be put to the fore for the sake of economics as science.

Although the mature form of Palomba's thought has *sui generis* tracts[9], its roots and scope are easily identifiable within the Italian Paretian School, whose main objective was to render dynamic the 'Paretian-type' competitive framework (Pomini and Tusset 2010). Palomba, pupil of Luigi Amoroso, is generally considered as a second-generation member of the School (Tusset 2004). At the roots of post-Paretianism

> the progressive abandonment of the evolutionary perspective, followed by strengthening of the static neoclassical method widened the interest in dynamic features. The logical outcome should have been dynamic equilibrium, but economists needed an instrument that would also enable them to explain the deep and rapid transformations affecting economic reality and productive systems from the nineteenth century to the twentieth century. The result was economic dynamics as an analysis of economic changes (Tusset 2009: 269)

[8] Mirowski (1986: 202) emphasizes the neglect of Palomba by the historians of economic thought, and places his name beside other eminent thinkers, like Ladislaus von Bortkiewicz and Benoît Mandelbrot.
[9] Palomba's early economic writings, despite being highly original, could be given a more orthodox interpretation (see Gandolfo 2008). For some biographical notes see Fusco (1986).

The need to address the issue of *change* in economic dynamics lies in the background of Palomba's theorizing. Palomba answers this need by providing an altogether new framework in which to account for economic change. The usual time-variable framework, i.e. an approach to modeling in which dynamics is dependent upon a time variable, is superseded by an axiomatic framework, in which dynamics is represented by an 'axiomatic stratification'[10] of distinct but interconnected theories (see Tusset 1998). The lapse of time is rendered through the switch from one axiomatic layer to the next, so that Palomba claims that "one can say that the time variable is absorbed into the spatial variable" (Palomba 1977: 1208)[11].

3.1 Natural philosophy and 'economic physics': the framework of multilayer axiomatization

Palomba's axiomatic program is pursued in the light of the conception of economic science as a *natural philosophy*[12] ("Anglo-Saxon style", as he explicitly claims). The evocative point of this label is easy to guess. For one thing, economics has to be driven by an 'experienced' reality: this is the "Anglo-Saxon style" character of his framework. But this is not all. Palomba further specifies that the meaning of his own *natural philosophy* has to be understood in a Galilean sense, and not in an Aristotelian one (Palomba 1977: 1187). This specification is meant to express the principle of *modern science* for which empiricism cannot be detached from a formal element that frames and gives structure to it.

Palomba, following in the footsteps of Pareto and Amoroso, is fascinated by treating economics on the same footing as physics. However, this may be more than an analogy for him: Palomba's work expresses the full acceptance of the Galilean point of view (see above) for which the 'book of nature is written in the language of mathematics'. The link with physics is construed, in fact, as we

[10] The expression is drawn from Palomba (1967a).
[11] The English translations from Palomba's Italian texts are mine.
[12] In English in the text.

are going to see, at the foundational level. That is, Palomba's interest does not consist simply in borrowing physical 'images' like motions, forces, equilibria and perturbations as metaphors, but rather in replicating the mathematical foundations of physics, which are geometrical in nature:

> [a]ccording to Souriau[13] <<the physical universe U is a geometrical universe>>, so that given a *set* E provided with a spatial structure such that any movement is recordable and referable to it, the set has a *universe-structure*. Further there must be an *operator* and a *field* in U that characterize its evolution in such a way that physical laws determine a family of *invariants* the field belongs to, so as to say that the latter, which constitutes 'sensible reality', can belong to a certain family of 'purely abstract possibilities'. To build such a physics, the properties of U, of the operator and of the family of invariants must be specified or defined. In such a way no one can prevent us from speaking of an 'economic physics' when we can set up such a framework (Palomba 1977: 1187-88, italics in the original)

From this dense quote, a series of fundamental features of Palomba's framework may be extracted:

- analogous to the assertion that "the physical universe U is a geometrical universe", Palomba's 'economic physics' poses that the "*economic universe* U is a geometrical universe". Abstractness of the geometric structure provides the logical possibility to build up different configurations within the abstract economic universe: Palomba claims that "pure economics, methodologically, is a genuine geometry", so that "from the logical point of view, there are as many pure economies as there are economic systems that may be thought of or tried out" (Palomba 1950: 253);

- different configurations of the abstract structure are modeled relying on different *metric spaces*, which define the geometrical characteristics of each universe (see also Palomba 1969b: 593-594);

- each configuration of the economic universe is meant to represent an economic *phenomenal field* (such as competitive states, monopoly, oligopoly, planned economy);

[13] Jean-Marie Soriau was a mathematician, pioneer in the fields of *differential geometry* and *differential topology*.

- each economic phenomenon is identified and expressed through a *group of transformations*[14]. The changes of metric in the abstract economic universe make arising the problem of performance comparison among economic units: this issue is resolved by identifying economic invariant principles, relying on them for comparison purposes. Within each economic phenomenon, and related group of transformations, such role is accomplished through the use of specific operators[15].

Through this analytical schema, characterized by the quadruple *economic universe – metric space – phenomenal field – group of transformations*, Palomba proposes four[16] layers of successive axiomatizations[17], each meant to encompass a different phenomenon of the economic domain. It can be useful to inquire closely into each of these layers, in order to see in which way Palomba's abstract schema is actually operationalized.

The first axiomatic layer (Palomba 1968) represents a *probabilistic* (economic) *universe*. The associated phenomenal *field* consists in the generalization of the Pareto-Amoroso framework of competition, so as to encompass, within a common axiomatization, the 'perfect and unperturbed

[14] Groups of transformations are defined as those transformations (over a set of objects) that share a common operator, and that satisfy: i) the property of closure in the composition of transformations; ii) the existence of identity and inverse transformations; iii) the associative property of transformations. For an introduction to groups of transformations in economics see Sato and Ramachandran (1998).

[15] To illustrate the notion of operator in the domain of economics Palomba proposes a very simple example, drawn from Paretian theory of general equilibrium. The theory of Paretian equilibrium can be conceived, in a stylized way, as a transformation that *maps* point-to-point any given initial quantity q into a final quantity q'. Think of an economic agent who possesses, in an initial state, n goods in the quantities $q_1, q_2, \ldots, q_n$ and that, if the n prices of the goods are known, sells and buys these goods according to a personal criterion of preferences a_{rs} (that expresses the willingness to exchange r for s). Assuming that the sum of exchange preferences is normalized to 1 and that $\sum_r a_{rs} = \sum_s a_{rs} = 1$, the transformation can be expressed as:

$$[q][a_{rs}] = [q']$$

which defines a group of transformations (if the requirements of note 15 are also met), according to the linear operator $[a_{rs}]$, which is a matrix that expresses the complete set of preferences of an agent, or to put it differently his exchange behavior (Palomba 1968: 770-771).

[16] In addition, the Paretian- and the Amoroso-type general equilibrium models represent a 0th axiomatic layer (see Table 1, below).

[17] Palomba proposes a technical illustration of the four axiomatizations in three articles that appeared in *Giornale degli economisti e Annali di economia* between 1968-1969 (Palomba 1968, 1969a, 1969b). Subsequently, under pressure "by more than one reader" Palomba provides a non-technical illustration of previous articles, in the same journal (Palomba 1970). The framework remains substantially untouched in later presentations.

competitive state', the 'perfect but perturbed competitive state' and the 'quasi-perfect and perturbable competitive state' (ibid.: 792-793). The shift from the Pareto to the Amoroso characterization of equilibrium carries with it a theoretical problem to be addressed, since – unlike Paretian static equilibrium – Amoroso-type dynamic equilibrium no longer guarantees that firm profits are zero at any point in time (Amoroso 1942). This acknowledgment gives rise – according to Palomba – to the need to tackle the problem of profit determination at the intra-firm level. This issue looks particularly troublesome since profit has to be determined according to the profit and loss account and the balance sheet, two reporting systems that are intrinsically different (all the more since they are both vulnerable to the subjective choices of the bookkeeper). To deal with profit determination, the *hermitian operator* Ψ is introduced (the same operator used in the Schrödinger equation to deal with the quantum mechanics problem of determining position and momentum of a particle). The role of the operator Ψ is to minimize the discrepancies in the probabilistic distributions of profit that emerge from the profit and loss account and the balance sheet (see also Palomba 1967b).

The second axiomatic layer (Palomba 1969a) represents an *inertial* (economic) *universe*. The associated phenomenal *field* consists in the market forms of monopolies and oligopolies. The emergence of systematic and persistent market disequilibria generates the theoretical problem of commensurability of economic performances for different economic units in the market. To solve the problem of commensurability, Palomba relies on the *Lorentz transformations*. He explicitly translates findings of Einsteinian special relativity – according to which accounts of space and time of two observers are, under certain conditions, specific to their reference frame – stating, analogously, that 'economic time' is not the same for different economic units (which are so assumed to act in different reference frames). According to Palomba, the flow of 'economic time' in each reference frame would be driven by a specific economic variable, i.e. the amortization speed of fixed capital. This variable would determine a sort of deformation of the space-time of the firm: the time shrinks (i.e. amortization speed is faster) for a more prosperous firm, but expands (i.e. amortization speed decelerates) for a worse-performing one. In order to provide a comparison among economic units in different ‘economic times’, appropriate corrections are needed, and can be obtained by Lorentz transformations. Analogous to the speed of light in special relativity theory, Palomba identifies an upper bound for the speed of amortization of firms. Importantly – still in accordance with special relativity – relativistic corrections become necessary for economic units when the speed of amortization approximates to its upper bound level; otherwise, usual (Galilean) Lagrangian equations apply as special cases[18].

The third axiomatic layer (Palomba 1969b) represents a *gravitational* (economic) *universe*. The related phenomenal *field* coincides with that of the preceding layer (i.e. monopolies and oligopolies) but it is meant to generalize it. It is devised, in fact, so as to encompass firms’ varying (either accelerating or decelerating) amortization speeds. A four-dimensional space is considered for each economic unit (each coordinate is meant to represent one of the four factors of production: labor, capital and land, plus time). The analysis chiefly aims at identifying the reference frame for

[18] It can be useful to provide a mathematical tasting of Palomba’s framework. In this axiomatic layer, for instance, Palomba relaxes the assumption that time is homogeneous among economic units: in production environments in which the amount of fixed capital is low, time flows according to the variable of the amortization speed of capital v. When the v of the firm is close to the <<modal>> v of the industry, the determination of the net profit y can expressed by the relation

$$y = Y - vt$$

where Y is the ‘gross’ profit. At this point, Palomba introduces the constant w, i.e. the value limit of the amortization speed of capital. When v is high enough, the ratio v/w is not negligible and the problem of commensurability among firms emerges. Relativistic correction are necessary to reach the correct value of Y and t. These corrections are provided by the *Lorentz* transformations (see Palomba 1969a: 185 for the ‘group’ considerations concerning the Lorentz transformations):

$$Y' = \frac{Y - vt}{\sqrt{1 - \frac{v^2}{w^2}}}$$

and

$$t' = \frac{t - \frac{v}{w^2}Y}{\sqrt{1 - \frac{v^2}{w^2}}}$$

As we can see, when v is negligible the ratio v/w is close to zero, and the profit determination collapses in the special case of the <<modal>> v among the firm of the industry. In better performing firms, $Y'>Y$ in a time $t'<t$. In the next layer, axiomatic layer 3, the assumption that v is uniform will be further relaxed.

each economic unit (Palomba considers in this layer mainly macroeconomic units as regions, nations, etc.), called ‘proper time of development’[19], and at providing relativistic corrections in order to make comparison feasible among these units. The operator associated with the gravitational space is a *metric tensor*, like that used in Einstein’s general relativity.

The fourth axiomatic level (Palomba 1969b) represents a *planning* (economic) *universe*. The associated *phenomenal* field consists of the phenomenon of economic planning. The intuition informing this axiomatic layer is that an economic plan is built within the constraints of an autonomously (steady-state) expanding economic universe. So, a planner, in order to make the plan feasible, should from the outset provide a synchronization of the calendar time with the ‘proper time of development’ of the planning unit. In other words, a proper ‘planning time’ has to be set. Accordingly, a suitable curvature has to be impressed on the economic universe in order to set the ‘planning time’. A ‘maximum planning time’ for time execution has to be identified in a range whose minimum is the ‘proper time of development’ itself and whose maximum is the limit of the amortization speed of capital. These corrections are made according to the relativity model proposed by the Italian physicist Luigi Fantappiè, by means of *Fantappié group*, also called *final relativity group*.

Table 1 provides a conceptual map of Palomba’s multilayer axiomatics.

[Table 1 about here]

By construction, as a fundamental characteristic of Palomba’s architecture that will be from now on widely emphasized, each subsequent axiomatic layer is devised as to encompass – mathematically

[19] Palomba (1970: 62) refers to the works of François Perroux, Ferenc Jánossy and Giovanni Demaria as forerunners of the notion of ‘proper time’ in economics. More recently, Mandelbrot re-presented a similar conceptualization of time, i.e. specific to each economic unit (see Mandelbrot and Hudson 2004).

and phenomenally – the previous one as a special case. As Palomba claims, using the emphatic words of the philosopher of mathematics Albert Lautman, his axiomatics aspires to represent an "ascension towards the absolute" (Palomba 1970: 48). In this way, Palomba importantly poses (e.g. 1968: 772) his framework of progressive generalization and abstractness of economic theories in direct connection with the 'Erlangen Program' of geometry.

3.2 Empirical progress *as* historical progress

While each axiomatic layer depicts a specific economic phenomenon (i.e. competition, monopolies, oligopolies and economic planning), it encompasses, as we have seen, the phenomena of the previous layers as special cases (i.e. economic planning is a generalization both of competition and asymmetric competition while, in turn, asymmetric competition is a generalization of perfect competition, and so on). In this sense, one can easily say that Palomba's axiomatics is meant to be 'empirically progressive', the 'retention' of previous phenomena being a fundamental ingredient of progressiveness.

However, Palomba is aware that the 'empirical progressiveness' of economic theories cannot be the same as that of physical theories. In particular, two substantial differences of the economic domain with respect to the physical domain have consequences for the status of economics as science. First, while physics "covers all the reality to be explained, economic physics covers only part of it. Part is left to the study of past as history, part to the creation of the future" (Palomba 1981: 165). Second and more importantly, humans, interacting in time, intentionally or unintentionally produce "new facts" (*e.g.* Palomba 1977: 1188) that are not explicable relying on already existing theories. The driver of economic theorizing so becomes precisely the attempt to give an account of continuously emerging 'new facts'.

The ‘new facts’ issue is indissolubly linked with the interpretation of economic dynamics as ‘change’. This is a characterization of economic dynamics that the Italian Paretian School recognized thank to the direction traced by Maffeo Pantaleoni (see e.g. Giocoli 2003). Not by chance, Pantaleoni (1907) was also the author of the attracting metaphor of ‘cinematograph’, used to represent the evolution of economic theories. It is under the lens of this metaphor – I argue – that Palomba’s axiomatics can be aptly read: the succession of axiomatic layers can be considered as the succession of frames of the ‘movie’ of economic evolution.

In this framework, the movie projected by Palomba is that of economic history. Following ‘new facts’ and accounting for them means, in other words, expressing precise historical judgments. In such context, Palomba states that the ‘Smith-Pareto’ world – that is, a world characterized by the phenomenon of competition – was not anymore actual. According to this historical situation, his zeroth axiomatic layer was insufficient, on the theoretical side, to cope with the new world and had to be supplemented. While the addition of probabilistic assumptions led Palomba to propose a general axiomatics for competitive states – where different parameter values are able to account for more complex forms of competition (first axiomatic layer) – this was not, once more, sufficient after the historical onset of monopolies and oligopolies (Palomba 1977: 1188) that “breaks up the ‘ergodicity’ of variables that compose the set under *observation*” (Palomba 1969: 795). New axiomatic layers are needed (so the second and third axiomatic layers). The same is true for the historical appearance of planned economies (fourth axiomatic layer). Palomba, we see, answers these historical challenges by providing more general axiomatic layers able to encompass them. Empirical progressivity is so conceived in this framework as historical progressivity. Economics, under this point of view , would be the discipline able to dissolve the constitutive tension between ‘history’ and ‘nature’, so that Palomba went so far defining the status of “*economics as a synthesis of history and nature*” (Palomba 1975a: 16, emphasis added).

To get a step deeper into Palomba's point of view, it would be useful to read it in the light of Joan Robinson's (1980) categories of *logical* and *historical* time[20]. 'Logical time' (i.e. the analytic time of a model) is, in Palomba's framework, not *longitudinal*[21] to the 'historical time' (i.e. the time of historical events), as it is usually the case in economic theories, but *transversal* to (split by) it. 'Logical time', in other words, is an analytic category that assumes meaning only within each axiomatic layer. Each axiomatic layer has its proper 'logical time': think of the axiomatic layers 2 and 3, in which time is respectively constant and variable (Palomba 1974). 'Historical time' is instead represented by means of the kinematic sequence of axiomatic layers (see Figure 1 below).

[Figure 1 about here]

To sum up, Palomba's conception of axiomatics is aimed at two simultaneous objectives: i) *empirical/historical representativeness* and ii) *retention* and *generalization*[22] of phenomena in progressive axiomatizations. The first objective is pursued by means of qualitative techniques of historical sensitivity; the second objective is pursued by means of the formal instrument provided by abstract geometry. In Palomba's words, this twofold objective of economic theorizing informs the status of economics as science:

[20] I use Robinson's categories although Palomba did not show particular sympathy for the Cambridge theories of that time (in this sense, he always remained a Paretian thinker). For Palomba and non-Paretian strands of coeval economic thought see Fusco (1986: 155-156).

[21] 'Longitudinality' has to be intended as the property of following the time as it lapses (e.g. the notion of 'longitudinal data').

[22] I will only address in this note (though it would deserve a more careful inquiry), the extent to which Palomba's framework is *technically* successful – according to the standards provided by himself. A preliminary look at Table 1 suffices to attest that Palomba does not succeed in ensuring that each axiomatic layer is a generalization of the preceding layer. For instance, if we consider the criterion of metric spaces, we can see that from a Riemann space, in the third layer, we go back to a more restricted Euclidean space at the fourth layer. Further, Palomba recognizes that a satisfying economic interpretation of technical devices borrowed from physics is not always attained (see Palomba, 1969b).

"an economic science exists if it can explain hard facts in front of it; an economic science exists if it can encompass such a reality in a geometry or transformation group from which to extract invariants and constants" (Palomba 1981: 164)[23]

4. Palomba's structural realism

The interpretative strategy of this essay, as stated from the outset, is to inquire whether – and to what extent – Palomba's original theoretical architecture is susceptible of a 'structural realist' interpretation, and, in case, to what extent this can be considered as an intellectual achievement for economic thought. A series of methodological problems – both of general and specific kinds – are connected to such an attempt. As Mäki rightly points out, for instance, the ontological commitment of theories may not coincide with the ontological beliefs of theorists: the impossibility of freely assuming that these ontologies coincide leads usually to considering theories as 'instrumentalist' products[24] (Mäki 2001: 10). Furthermore, it is difficult to find, in economic texts, expressions that unambiguously (i.e. not naively) express an ontological commitment. Still, in our case, Palomba's intellectual *milieu* was not acquainted with the English-speaking world's philosophy of science categories, so that it is imprudent even to guess whether Palomba could have been aware of the possibility of being realist in the current sense[25]. Nonetheless, it will be the task of this essay to provide a 'structural realist interpretation' of Palomba's thought using contemporary categories – as far as it seems useful. Such exercise would be useful in fact to the extent that it will allow us to introduce and prove a new interpretative category in the history of economic thought.

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[23] I argue that far from isolated and idiosyncratic, Palomba's conception of economic science and of the best tools for pursuing it was shared by other Italian economists of the time like Giovanni Demaria. Palomba credits Demaria for inspiring the "reduction of economic phenomena […] to abstract algebra" (Palomba 1968: 794-795). Giovanni Demaria in some late writings (e.g. 1981) explicitly poses the fundamental question of the relationship between "symmetries and political economy". For his part, however, Demaria was clear in stating that the general notion of symmetry should not be identified with that of structures, basically for the simple reason that not all structures are symmetric (Demaria 1981 534).

[24] The champion of the instrumentalist interpretation of economic theories is Friedman (1953).

[25] Hands (1990: 74) claims that there is consensus in economic methodology that the founders of neoclassical economics were 'realist' in the current sense of the term. This means that 'realism' is a sort of default interpretative heuristics for the assessment of early economists' theories.

Section 2 provided an account of what is broadly meant by ‘structural realism’, while Section 3 provided a sketch of Palomba’s axiomatics. The thesis of this essay is that – even before considering the possibility of structural realism in economics – Palomba endorses structural realism in physics. This is reasonably implicit in his endorsement of the conception of physics as founded on an abstract ‘geometrical universe’: within the logical structure of the ‘geometrical universe’ Palomba then looks at the succession of physical theories as following an approach either of *retention of structures* or of *generalization*. It is this framework that he basically wants to import into economics[26].

But let me go step by step. At a first level of approximation, all the various attempts to look at physics as a repository of structures to be employed in economics are, *potentially*, ‘structural realist’ attempts. Think of the emerging field of econophysics and of its more or less implicit structuralist assumptions: it does not matter whether the objects of predication are different between physics and economics (rocks and tables on the one hand, consumers, firms and states on the other), since what hold true in both domains are the same mathematical structures of theories. We find, clearly, this point of view in Palomba too[27]. Such attitude toward theories expresses a point of view that we have already met in Section 2, that is the ‘semantic’ or ‘model-theoretic’ conception of theories. As we have seen, such view of theories is usually considered to be a requirement conducive to the structural realist hypotheses.

But clearly, there is something more that sustains this essay’s view of Palomba as a structural realist. Palomba’s distinctive use of groups of transformations is the best way to start from. The mathematics of groups of transformations constitutes more than a mere technical device in his

[26] Reporting the words of the German astronomer August Kopff, Palomba expresses his point of view on physics in this way: “Physics [hat to be] intended in the widest sense of the word; so not inquire of specific phenomena of nature or of experience and search for their explanations, but comprehension of ALL THE PHYSICAL PHENOMENA FROM A UNITARY POINT OF VIEW” (Palomba 1952: 57, italics in the original, capital letters added; reported in Tusset 2004: 149).

[27] Palomba, not by chance, can be considered one of the fathers of econophysics for his early studies on the ‘index of the degree economic complexity’ (Palomba 1939).

framework (as, for other economists, calculus may be). As far as this essay is concerned, it rejects the idea that any use whatsoever of groups of transformations in economics would suffice to prove that we are dealing with 'structural realism'. That would be too simplistic, and misleading. In a genuinely 'structural realist' perspective, groups of transformations should play a deeper role: they should identify relevant structures, and then they should be considered as bridges to establish continuity among theories and/or phenomena. In Palomba's axiomatics, groups of transformations play in fact this twofold role. The primary role is that each group of transformations is associated with an axiomatic layer (which expresses a phenomenal field), and in this way – as we have seen – is able to compare *all* varieties of phenomena at that layer. For instance, for the second axiomatic layer, the Lorentz transformations are able to make comparable each pair of economic units with different (but constant, otherwise the third layer applies) speeds of amortization. In the same axiomatic architecture, however, the groups of transformations at different axiomatic layers are not mutually independent. A link connects those layers – this link being the second role played by groups of transformations: it consists in the 'higher degree of generality' of the group at the next axiomatic layer with respect to the group at the previous one. Palomba establishes a hierarchy among axiomatic layers, by means of the sequential 'generality' of groups of transformations. When economic phenomenal world changes, with the arrival or prevalence of monopolistic, oligopolistic, planning or whatsoever phenomena, so that "*the current group does not encompass all phenomena to be studied or to be theorized* […] the central point is what follows: […] *it is necessary to rely on a wider group that includes the latter as a special case*" (Palomba 1977: 1207, italics in the original)

Summarizing, I claim that Palomba conceives groups of transformations in a twofold way: not only *within* each axiomatic layer to exhaust the empirical variety at that level, but also *between* axiomatic layers to establish empirical progress – conceived as historical progress – by means of retention of structures and generalization.

But what is it exactly that is retained and generalized in Palomba's framework? What is the underlying invariant he is so eager to retain? To answer such a question, we have to invoke Palomba's never suppressed Paretian orthodoxy. The economic invariant consists in fact in the notion of 'equilibrium', and in the related 'marginalist principle' that informs equilibrium. As Palomba clearly states:

our aim is just to reach, by means of suitable axiomatizations, more and more general views of Paretian economics to which correspond more universal extensions of the concept of equilibrium and of the related fundamental invariant, constituted by the marginalist principle and by the ophelimitarian maximum deduced from it (Palomba 1970: 47)

Such invariants are by definition ubiquitous, as we have already seen, both within each axiomatic layer and between axiomatic layer, i.e. in each form of conceivable economic system :

the marginalist principle […] is an invariant property […] not only of any equilibrium at any level of (reasonable) axiomatization; but also of any (reasonable) economic system, be it individualistic or socialistic in nature (ibid.: 49, n.10)

However, during the development of his ambitious 'pan-marginalist' economics, Palomba clashes with the difficulties inherent to the interpretations of different mathematical structures as bearers of a common semantic content (i.e. the marginalistic principle). As he warned, in an early passage, "the invariant properties being discussed are what they are only if conceived from a purely formal point of view, while if one inquires into their content even a little, a series of related contradictions begin to appear" (Palomba 1950: 337). Twenty years later, the conclusion of Palomba's inquiry consisted in saying that "these generalizations do not at all imply that equilibrium and the marginalist principle continue to mean the same things that they meant and still mean in the Paretian universe" (Palomba 1970: 48)[28].

[28] Palomba's axiomatics looks consistent with the 'formalist' methodological standard set out for economics in the late 1950s by Gerard Debreu, according to which "an axiomatized theory has a mathematical form that is *completely separated* from its economic content. If one removes the economic interpretations […] its bare mathematical structure must still stand" (Debreu 1986: 1265, emphasis added). It is interesting to note, for the argument of this paper, that Bob Clower's refusal of Debreu's methodology mirrors Psillos's arguments against structural realism, since

Although substantial parts of Palomba's axiomatics are consistent with a structural realist interpretation, it is worth considering some traits that do not fit so well with the literature on structural realism. It is interesting to see whether these dissonant traits would be detrimental to the structural realist interpretative thesis of this essay or whether, on the contrary, they would give useful insights for the hypothesis of structural realism in a cross-domain or cross-disciplinary case. First, as we have noticed, Palomba places a decisive emphasis on the issue of sequential *generalization* of axiomatic layers. The literature on structural realism has often considered cases in which retention of structure in theory change has proceeded by means of assimilation and generalization of previous theories (think of the shift from Newtonian/Galilean physics to Einsteinian physics). However, one may argue that generalization is not a necessary requirement of structural realism. One may argue that continuity and/or retention of structure may be accomplished in different ways that do not necessarily involve sequential generalization.

There is also a second, fundamental point. Consider those structuralist thinkers according to whom structural realism is a hypothesis that has to be established by looking primarily at the historical sequence of theories. In this sense, structural realism is conceived as a *re-constructive* enterprise, i.e. an *ex-post* inquiry into the invariant properties of the succession of historical theories. Palomba, instead, conceives structural realism in a *constructive* sense, structural invariance being taken as an *ex ante* constraint of theorizing.

The *constructive*[29] side of Palomba's theorizing is surely the aspect that is more prominent at a first glance. Nonetheless, a couple of remarks are in order, to underline how Palomba's thought is not devoid of re-constructive stimuli. First, we have seen that at the roots of his axiomatic attempts was the economic historian's acknowledgement that the phenomenal domain had been changing. Using

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Clower denies that the detachment between mathematical and economic content can be attained, at least not in the Debreu's framework.

[29] The label 'constructive' is here intended with no connection with the philosophical anti-realist position of Van Fraassen (1980) labeled 'constructive empiricism', although it surely makes arising the question whether a constructive attempt can be claimed to have realist value (for the discussion of this important theme, see below).

Palomba's label, the "Smith-Pareto" world was over. Given this historical context, Palomba then assumes that also the theories of Smith and Pareto were over. The practice of linking each historical period with *its own* representative theory may be read as a proper reconstructive activity, and a sort of prerequisite to constructive theorizing. In a second sense, still, the work of Palomba is extremely attentive towards re-constructive considerations. Palomba the 'historian of economic thought' reconstructed the history of economic thought trying to identify lineages among theories on the basis of structural considerations (see Palomba 1967a). For instance, he tried to reconcile perspectives that seemed incommensurable in the history of economic analysis, as those of Pareto and Marx, by eliciting some structural similarity (see Palomba 1973). However, even prior to be a historian of economic thought Palomba shows himself as a 'structuralist' historian of physics: the sequence of mathematical theories that he takes as underlying structure of his economic axiomatic layers is nothing more than a sequence of the most important physical theories of the 20th century, selected by virtue of their structural continuity/permanence. In all the three senses sketched above, Palomba's re-constructive structuralism looks like a precondition of his constructive structuralism.

On the other hand, the purely reconstructive interpretation of structural realism, as has been repeatedly emphasized by philosophers, is untenable as genuinely philosophically 'realist' position (see e.g. Psillos, 2001). The simple reconstruction and identification of structural invariances in theory change could be considered as an *ad hoc* procedure (for *adhocness* as major risk in structural realist positions see e.g. Ladyman and Ross 2007: 67). To avoid such risk, Worrall claimed that structuralism can gain the realist status only when the identified structures can be employed forward, to attain "*correct empirical predictions*" (Worrall 1989: 101, emphasis added). This is a central point that deserves particular emphasis in the cross-disciplinary structural realist story we are telling here. As the theoretical attempt of Palomba shows, structures that were previously employed to explain physical phenomena are now employed to explain economic phenomena, a domain, that of the economy, that was *unintended* as domain of analysis when those structures were originally devised in physics. Previously unintended cross-disciplinary application of theories is included among the many (controversial) senses in which a theory is considered to cope with 'novel fact' (see Murphy 1989). While it is not necessary to fasten Palomba's structural realist jacket so tight – since what this essay wishes to do is to suggest insights and material for further reflection, not to give answers – it is nonetheless important to remark that Palomba had very clear in mind that the structures of physical theories are devices that allow economists to inquire into 'unexplored' economic facts:

> the problem is to see whether and to what extent [operators employed in physics] are able to explain [economic] facts that are still left or *unexplored* or unexplainable (Palomba 1977: 1208, emphasis added)

5. Concluding remarks

The reconstruction of the work of Giuseppe Palomba allows to draw some concluding remarks. The first one concerns the meaning of 'heterodoxy' as a way to describe the character and extent of Palomba's thought. As is easily arguable – but hardly really acknowledged – heterodox thought is always meant to fill some gap in the orthodox rival. In our case, this essay claims that Giuseppe Palomba took more seriously than orthodox economists the relationship between economics and physics. As pointed out by Mirowski

> whereas in physics there was a continuous research program whose purpose was to come to understand the nature and limits of the new reified invariant [i.e. energy], it seems not one of the neoclassical economists even understood what was at stake in the problem of invariance (Mirowski 1989: 271).

Palomba instead put right at the center of his analytical focus the issue of invariance in economics, mainly conceived of as structural invariance or continuity, inquired for the first time through the advanced tools of group theory[30].

[30] Tusset (1998) does justice to Palomba, acknowledging his primacy in the introduction of the mathematics of transformation groups in economics. Previously, Samuelson had wrongly attributed the honor of the pioneer to Sato (in Sato 1981, preface). Mirowski (2012) claims that Bob Clower used group theory in an unpublished paper of 1967.

The second major remark concerns the interpretation of this focus on structural invariance as an example of 'structural realism'. In a sense, although for Palomba the answer to such question has been mostly positive – given all the *caveat* of the case – the issue of structural realism conveys an importance that is greater than the case of Palomba itself. It involves two important issues: i) whether structural continuity and invariance of theories is considered an important issue in economics; ii) the way in which economists consider the status of physical theories as employed in economics (an old issue indeed). The aim of this essay has been to boost further inquiry either into the figure of Palomba or into structural realism in the field of economics, in a similar vein, hopefully, in which a study of Elie Zahar (1994) on Poincaré as pioneer of structural realism boosted the study of those topics in the field of mathematics and physics. With all the due differences, under this very point of view, mathematical economics could have find in Palomba its own Poincaré.

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Table 1: A schema of Palomba's 'axiomatic stratification'

	Economic Universe	**Metric Spaces**	**Operator/ Transformation Group**	**Phenomenal Field / Comparison problems emerged and solved through transformations at each level**
Zeroth axiomatic layer	Stationary/ Inertial	Euclidean	Linear	Competitive states as conceived by Stationary Paretian Economic General Equilibrium and Dynamic Amoroso-type Economic General Equilibrium
First axiomatic layer	Probabilistic	Pseudo-Euclidean	Hermitian Operator Ψ	Competitive states (generalized)/ Profit determination at the intra-firm level
Second axiomatic layer	Inertial	Pseudo-Euclidean	Lorentz Transformations	Monopolies and Oligopolies/ Commensurability among economic units (at a fixed speed of amortization)
Third axiomatic layer	Gravitational	Riemannian	Metric Tensor	Monopolies and Oligopolies/ Generalized commensurability among economic units (variable speed of amortization)
Fourth axiomatic layer	Planning	Pseudo-Euclidean	Fantappié Group/ "Final Relativity" Group	Planning Phenomena / Reconciliation among calendar time, 'proper time of development' and 'plan time'

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Figure 1**:** The Historical time lapse (Ht) is represented, in Palomba's framework, through the succession of axiomatic layers. Each axiomatic layer has its own Logical time (Lt)