Local-average games

• This paper considers a tax evasion game between \( n > 1 \) individuals and the tax authority, who seeks to maximize the aggregate fiscal revenues collected from individual tax payments.

• It is assumed that taxpayer communication happens truthfully and voluntarily (Andrei et al., 2014), and where individuals assimilate the average value of the new information received from their neighbors (Hokamp & Pickhardt, 2010).

• The presence of social interactions leads taxpayers to experience peer effects (Fortin et al., 2007; Alm et al., 2017).

• The local-average or linear-in-means model is the workhorse model in empirical work on peer effects (Blume et al., 2015; Kline & Tamer, 2019; Ushchev & Zenou, 2020).
Taxpayer network $g$

- Consider $\mathcal{N}$ a set of $n > 1$ taxpayers which coexist in a connected network $g$, with a $n \times n$ adjacency matrix $H = [h_{i,j}]$ with entries $\{0, 1\}$, where $h_{i,j} = 1$ if and only if there is a direct connection between agents $i$ and $j$; otherwise $h_{i,j} = 0$.
  - The network is undirected and does not include any self-loops.
  - We say two agents or taxpayers are ‘neighbors’ if they share a direct link between each other.

- Define $G = [g_{i,j}]$ with entries $[g_{i,j}] \in [0, 1]$ as the $n \times n$ row-normalized adjacency matrix obtained from diving each entry of matrix $H$ by the degree of node $i$. Hence, $[g_{i,j}] = [h_{i,j}]/N_i$, where $N_i$ represents the node-degree of taxpayer $i$.
  - One can interpret the value of $[g_{i,j}]$ as the influence which agent $j$ exerts on agent $i$, in the sense of Degroot (1974).
Local-average games: utility function

- Local-average games (Blume et al., 2015; Kline & Tamer, 2019) have a linear-quadratic utility function of the form:

\[
U_i(x_i, x_{-i}, g) = \alpha_i x_i - \frac{1}{2} x_i^2 - \frac{\theta}{2} (x_i - \bar{x}_i)^2, \tag{1}
\]

- \(x_i\) is the outcome (e.g. tax payment) exerted by agent \(i\),
- \(x_{-i}\) is the vector of outcomes exerted by all other players,
- \(g\) is the social network,
- \(\alpha_i > 0\) is an individual *productivity* parameter,
- \(\theta\) is the *social interaction effect* which measures an agent’s reaction to the average outcome of its neighbors (e.g. *alla romana*).
- \(\bar{x}_i\) is the individual-specific *social norm*, defined as the average outcome exerted by agent \(i\)’s neighbors weighted by the influence exerted by each player \(j \neq i\) on taxpayer \(i\). Namely:

\[
\bar{x}_i = \sum_{j=1}^{n} g_{ij} x_j. \tag{2}
\]
A quick look from the taxpayer’s perspective

• Assume a taxpayer’s value function (or expected utility) is:

\[ V = \hat{p} \cdot \nu(\text{audited}) + (1 - \hat{p}) \cdot \nu(\text{not audited}). \]  

\[ (3) \]

• The generalized taxpayer’s problem is to maximize the value function \( V \) in terms of the payoffs \( \nu \) of being audited or not:

\[ \max_{\{d\}} V(\hat{p}, d, I, \tau, \phi, \cdot) \]  

\[ (4) \]

where agents optimize only over the declared income \( d \). Notice that the subjective audit rate \( \hat{p} \) is endogenous; while income, taxes, penalties and most other parameters are exogenous.

• A higher perceived audit rate, \textit{ceteris paribus}, would induce taxpayers to be more compliant (Casal & Mittone, 2016).

  • Optimal \( d \) is (weakly) \textbf{increasing with respect to} \( \hat{p} \).
  • Hence, tax payments \((x_i) \propto\) declared income \((d_i) \propto \hat{p}_i\).
Mathematically equivalent problems

- Define individual tax payments as \( x_i := d_i l_t \tau \)
  - \( d_i \) is the individual fraction of income disclosed,
  - \( l_i \) is the taxpayer’s exogenous given income,
  - \( \tau \) is the societal tax rate (flat or stepped).

Claim (1)

From the point of view of the tax authority, in a local-average game, the two problems are mathematically equivalent:

\[
\text{argmax}_A \ e^\top x = \text{argmax}_A \ \frac{1}{n} e^\top \hat{p},
\]

where \( A \) is the set of possible actions of the tax authority (e.g. audit probabilities, sequence of audits, targeted audits, etc.).

\( x := (x_1, x_2, \ldots, x_n)^\top \in \mathbb{R}_+^n \) is the vector of tax payments and \( \hat{p} := (\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_n)^\top \) is the vector of subjective audit rates of all players in network \( g \), and \( e \in \mathbb{R}^n \) is a column-vector of ones.
Local-average games: Nash Equilibrium

• We redefine the local-average game in terms of $\hat{p}_i$ as:

$$U_i(\hat{p}_i, \hat{p}_{-i}, \varphi) = \alpha_i \hat{p}_i - \frac{1}{2} \hat{p}_i^2 - \frac{1}{2} \left( \frac{\lambda}{1 - \lambda} \right) (\hat{p}_i - \bar{\hat{p}}_i)^2,$$

(5)

• where $\theta = \frac{\lambda}{1 - \lambda}$ and $0 < \lambda < 1$.

• The best-reply function for each taxpayer $i$ is given by:

$$\hat{p}_i = (1 - \lambda)\alpha_i + \lambda \bar{\hat{p}}_i,$$

(6)

Proposition (1)

Solving for $\hat{p}$ the Nash Equilibrium ($\hat{p}^*$) is defined by:

$$\hat{p}^* = (1 - \lambda)[I - \lambda G]^{-1}\alpha.$$
Local-average games: Heterogeneity

Proposition (2)

The matrix $M := (1 - \lambda)[I - \lambda G]^{-1}$ is well-defined and row-normalized for any $\lambda \in (0, 1)$. Hence one has: $\hat{p}^* = M\alpha$.

Proposition (3)

Since $G$ is a row-normalized adjacency matrix, the Nash Equilibrium exists, is unique and is interior for any $\lambda \in (0, 1)$.

Claim (2)

If individuals are ex ante homogeneous, that is if $\alpha_i = \alpha_j$ for all $\{i, j\} \in \{1, 2, ..., n\}$, then the aggregate and individual Nash Equilibrium outcome levels will be independent of the network structure, rendering network-based policies useless.
Threat-to-audit message

- Threat-to-audit messages can affect taxpayer behavior (Boning et al., 2018; Lopez-Luzuriaga & Scartascini, 2019).

**Tax authority’s message:**

Dear citizen,

A new audit regime is in place. Last year the societal audit probability was of $p$ and equal for all taxpayers. As of now, the probability of being audited will be proportional to the income level of each taxpayer. Hence, the individual-specific audit rate for each taxpayer $i$ is now defined as:

$$p_i = p \cdot \frac{l_i}{\sum_{j=1}^{n} l_j} \cdot n,$$

(7)

where $p$ is the homogeneous true audit rate from last year, $l_i$ denotes the gross earned income of taxpayer $i$, $n$ is the total number of individuals in the society, and $p_i \in [0, 1]$ for all $i \in \{1, 2, ..., n\}$.

- The average and aggregate probabilities have not changed, just shifted.
Ensuring taxpayer *productivity* heterogeneity

- Following the threat-to-audit message, taxpayers compute their income heterogeneity with respect to society.

- Let $\alpha_i$ be determined by an agent’s income divided by the average income of all the agents in the network. The value of such individual-specific heterogeneity level $\alpha_i$ is defined as:

$$\alpha_i = \frac{l_i}{\sum_{j=1}^{n} l_j} \cdot n$$  \hspace{1cm} (8)

- The interpretation of $\alpha_i$ would be a taxpayer’s exogenous-given income *productivity* with respect to society.
  - E.g. if $j$’s income is twice the average income level, then $a_j = 2$.

- Averaging on both sides, it is easy to see that the average and aggregate productivity in the network have not been modified.
Subjective probability of being audited

- Individual **belief dynamics in tax compliance** are strongly path-dependent with respect to the average past behavior of other players (Alm et al., 2017; Gächter & Renner, 2018).
- In general, subjective audit rates may be affected by **three channels**: prior beliefs, empirical audit rates and the socially-learned value of the audit rate in its neighborhood.
- In a dynamic framework, the endogenous and **post-message** heterogeneous subjective audit rates can be formulated as:

\[
\hat{p}_{i,t+1} = \frac{1 - \omega}{2} \hat{p}_{i,t} + \frac{1 - \omega}{2} \frac{1}{m} \sum_{s=1}^{m} A_{i,t-s} + \omega (\alpha_i \tilde{p}_{i,t}), \quad (9)
\]

where \( \omega \in (0,1) \) is the weight given to the newly acquired information, \( A_{i,t-s} = 1 \) if agent \( i \) was audited at time \( t - s \) and zero otherwise, and \( \alpha_i > 0 \) is the income productivity level.
First-best outcomes and restorations

• Local-average game first-best outcomes and restorations are well-defined (Ushchev & Zenou, 2020).

**Proposition (4)**

*Given a local-average game as previously characterized, the first-best outcome, $\hat{p}^o$, is a solution to:*

\[
\hat{p} = (1 - \lambda)\alpha + \lambda G\hat{p} + \lambda G^\top (I - G)\hat{p},
\]

*whose solution is unique, and it is given by:*

\[
\hat{p}^o = \left[ I + \frac{\lambda}{1 - \lambda} (I - G)^\top (I - G) \right]^{-1} \alpha.
\]

• The first-best outcome is expressed in function of the productivity ($\alpha$), taste for conformity ($\lambda$) and network structure ($G$).
First-best outcomes and restorations

• When the players in a local-average game do not reach the first-best equilibrium, the social planner (tax authority) may try to restore it by subsidizing or taxing specific individuals.

Proposition (5)

The first-best outcome is restored when the social planner endows agents with the following subsidy/tax per unit of effort:

\[
S^o = \frac{\lambda}{1 - \lambda} G^\top (I - G) \hat{p}^o,
\]

where the optimal per-effort subsidy for each agent \( i \) is:

\[
S_i^o = \frac{\lambda}{1 - \lambda} \sum_{j \neq i} g_{ji} (\hat{p}_j^o - \bar{\hat{p}}_j^o).
\]
Maximizing the aggregate outcome

- The objective of the tax authority is to audit the set of taxpayers, $\mathcal{M} \subset \mathcal{N}$, such that the global subjective audit probability is maximized, and constrained by a finite number of audits $\lfloor np \rfloor$.

\[
\max_{\{\mathcal{M} \subset \mathcal{N}\}} \frac{1}{n} \sum_{i=1}^{n} \hat{p}_{i,t+1}(A_{i,t}, A_{-i,t}, \cdot)
\]

\[\text{s.t. } A_{i,t} = 1 \iff i \in \mathcal{M}, \]

\[A_{i,t} = 0 \iff i \notin \mathcal{M}, \]

\[|\mathcal{M}| \leq \lfloor np \rfloor, \tag{10}\]

where the individual subjective probability for all taxpayers at time $t + 1$ is dependent on whether they have been audited or not ($A_{i,t}$), and on who else was audited or not ($A_{-i,t}$).

- The **solution of the tax authority’s problem** is to compute the vector of optimal individual subsidies ($S^o_i$) and to audit the $\lfloor np \rfloor$ taxpayers with the maximal individual subsidy values.
Taxpayer simulation

• Let us define a _dynamic game_ in a taxpayer network with social interactions. First, agents and society are characterized and the social network is built. Then, the tax authority emits a message to incentive tax compliance.

• Each period, agents disclose a share of their income, may or may not be audited, and then exchange information with their neighbors and update their subjective audit rates.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>Agents (taxpayers) are parameterized.</td>
</tr>
<tr>
<td>Step 2</td>
<td>The social network is built.</td>
</tr>
<tr>
<td>Step 3</td>
<td>The tax authority emits a threat-to-audit message.</td>
</tr>
<tr>
<td>Step 4</td>
<td>Agents hold social interactions and share information.</td>
</tr>
<tr>
<td>Step 5</td>
<td>Agents choose their optimal declared income.</td>
</tr>
<tr>
<td>Step 6</td>
<td>The tax authority applies its optimal audit strategy.</td>
</tr>
<tr>
<td>Loop</td>
<td>Go back to Step 4.</td>
</tr>
</tbody>
</table>
Taxpayer characterization

- Social networks of tax evasion consider **homophily** behavior and **cohesive** relations among individuals (Andrei et al., 2014; Gamannossi degl’Innocenti & Rablen, 2020). That is, taxpayers tend to form links with peers who are akin to them and with whom they share similar traits and characteristics.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Exog.</th>
<th>Endog.</th>
<th>Societal</th>
<th>Individual</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l ): Earned income</td>
<td>( X )</td>
<td>( X )</td>
<td></td>
<td>( X )</td>
</tr>
<tr>
<td>( \tau ): Tax rate</td>
<td>( X )</td>
<td>( _ )</td>
<td>( X )</td>
<td>( _ )</td>
</tr>
<tr>
<td>( \phi ): Penalty rate</td>
<td>( X )</td>
<td>( _ )</td>
<td>( X )</td>
<td>( _ )</td>
</tr>
<tr>
<td>( m ): Fiscal memory length</td>
<td>( X )</td>
<td>( _ )</td>
<td>( X )</td>
<td>( _ )</td>
</tr>
<tr>
<td>( n ): Number of taxpayers</td>
<td>( X )</td>
<td>( _ )</td>
<td>( X )</td>
<td>( _ )</td>
</tr>
<tr>
<td>( \omega ): Weighting parameter</td>
<td>( X )</td>
<td>( _ )</td>
<td>( X )</td>
<td>( _ )</td>
</tr>
<tr>
<td>( \theta ): Taste for conformity</td>
<td>( X )</td>
<td>( _ )</td>
<td>( X )</td>
<td>( _ )</td>
</tr>
<tr>
<td>( p ): True audit rate</td>
<td>( _ )</td>
<td>( X )</td>
<td>( X )</td>
<td>( _ )</td>
</tr>
<tr>
<td>( \hat{p} ): Subjective audit rate</td>
<td>( X )</td>
<td>( _ )</td>
<td>( X )</td>
<td>( _ )</td>
</tr>
<tr>
<td>( d ): Declared income</td>
<td>( X )</td>
<td>( _ )</td>
<td>( X )</td>
<td>( _ )</td>
</tr>
<tr>
<td>( q ): Global subjective audit rate</td>
<td>( X )</td>
<td>( _ )</td>
<td>( X )</td>
<td>( _ )</td>
</tr>
</tbody>
</table>
Comparing audit strategies: convergence levels

- The proposed *Subsidy* strategy secured the highest average convergence level over 100 simulations per audit scheme.

**Figure:** Convergence level of the global subjective audit rate for different audit schemes: *Subsidy* (S), *Degree* (D), *Random* (R), *Intercentrality* (I), *Betweenness* (B), *Closeness* (C) and *Eigencentrality* (E).
Comparing audit strategies: outcome distributions

- The proposed *Subsidy* strategy obtained the highest convergence level distribution of the global (average) subjective audit rate at a 0.001% confidence level.

**Figure**: Distributions of the convergence levels of the global subjective audit rate for diverse audit strategies.
Testing parameter effects

(a) Number of taxpayers (n)

(b) Expected node-degree (μ)

(c) Constant density (δ)

(d) Attention to neighbors (ω)
Assessing robustness

(a) Initial subjective audit rate

(b) Taste for conformity ($\lambda$)

(c) Fiscal memory ($m$)

(d) Societal true audit rate ($p$)
Model extensions

- The proposed audit scheme would outperform random auditing and most policies if at least 35% of the links would be known.
- The tax authority could fully enforce the proposed optimal audit strategy if at least 70% of the links would be known.

(a) What if the tax authority cannot observe link directions?  
(b) What if the tax authority’s omniscience is limited?

Figure: Which would be the cost of discovering all taxpayer links?
Limitations of network-based strategies

- If taxpayers do not pay **attention** to the threat-to-audit message they will not be *post-message* heterogeneous ($\alpha_i = \alpha_j$).
- If the taxpayer network lacks all **cohesiveness**, the strategy would be useless. Fortunately, social networks are cohesive (McPherson et al., 2001; Moody, 2001; Currarini et al., 2009).

(a) Attention placed to the threat-to-audit message

(b) Cohesive and non-cohesive taxpayer networks

**Figure**: Graphical representation of the two model limitations.
Policy implications and concluding remarks

- This paper proposes a two-step game-theoretic optimal audit strategy from the point of view of the tax authority.
  - First step: Credible threat-to-audit message.
  - Second step: Network-based audit policy.

1. The proposed enforcement regime targets taxpayers in function of their *productivity* and their position in the network.

2. To the best of my knowledge, it is the first audit strategy that is robust to individual and societal parameters, such as:
   - Number of taxpayers, network density, true audit rates...
   - Taxpayer heterogeneity: attentiveness, memory, endogenous $p$.
   - Expected and Non-expected utility theories.
   - Invariant to any plausible utility and payoff functions.

3. Notwithstanding, the costs and plausibility of observing a given fraction of taxpayer links remain open questions.
References I


References II


References III


References IV


References V


References VI


