Seller competition and platform investment in two-sided markets

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Abstract

While most of the existing literature on platforms has focused on the issue of platform pricing strategies in presence of positive between-group external effects, our paper focuses on contexts where within-group effects also exist and non-price strategies are important. In particular, we develop a model where a monopolistic platform can invest in quality improvement, while levying transaction fees to sellers. We find that when the quality-improving technology is sufficiently productive, both platform and sellers’ profit are increasing in the number of sellers, i.e. the within-group external effects affecting sellers are positive notwithstanding competition. We show that this may occur both when competition is à la Cournot and à la Bertrand.

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1 Introduction

Platforms are ubiquitous both in the economy and in the IO and strategic management literature. In the business world, a series of innovative services that appeared in the last decades, such as retail and marketplace services, mobile commerce, customer service, e-procurement and purchase-to-pay, are commonly traded online by economic platforms operating in internet or in social networks. Beside e-commerce businesses (eBay), several creative industries such as Google (search engine), Facebook (social network), and Sony (gaming) as well as the modern cultural and tourism industries (Netflix, Spotify, YouTube, Booking.com, Expedia, TripAdvisor etc.) produce and distribute their contents by open or closed platforms. Not only platforms themselves are among the most innovative and successful start-up initiatives in recent years; but, more and more, both established businesses and new entrepreneurial ventures rely on them for their growth and performance.

The organizations that intermediate the interactions between two (or more) “customers’ are called multi-sided platforms and they form the so-called two-sided markets. As for the definition given by Rochet & Tirole (2003), a two-sided market is made of at least a platform in which sellers and buyers meet and trade, and has a volume of transactions that varies as the fees applied by the platform to the seller or to the buyer change while maintaining the aggregate fees constant. In other words, if the sellers’ fees increase (and hence buyers’ decrease), the volume of the transactions varies in a two-sided market; in case the volume would not change, we would have a one-sided market.

Most of the existing literature has focused on the issue of platform pricing strategies in presence of positive between-group external effects, i.e. when a larger number of sellers attract more buyers, and vice versa. Our paper instead focuses on contexts where within-group effects also exist and non-price strategies are important. As for the former point, we follow Belleflamme & Peitz (2019) by considering explicitly the competition among sellers. Platforms such Amazon and Booking.com are obvious examples, among many, of platforms in which sellers compete. As for non-price strategies, we consider platform investment that can increase buyer utility irrespectively of the specific seller that the buyer buys from. Again, opportunities for such quality-improving investment are observed for many platforms. For instance, companies such as Sony and Apple may improve the technological features of their consoles and phones which
may advantage consumers in the use of any videogame or app.

In this paper we develop a model where a monopolistic platform can invest in quality improvement, while levying transaction fees to sellers. In the current version of the paper we consider an exogenously given number of sellers and buyers, and we focus on the sign of within-group effect for sellers. On one hand, an increase in the number of sellers increases competition, and so it reduces sellers’ profit. This the standard negative within-group external effect, discussed in Belleflamme & Peitz (2019). However, a larger number of sellers makes the investment by the platform more profitable. It turns out that when the quality-improving technology is sufficiently productive, the second effect prevails, and the within-group external effects affecting sellers turn positive. We show also that this may occur both competition is à la Cournot or à la Bertrand.

This paper is organized as follows. Section 2 provides a literature review, and locates our contribution within the existing research. In Section 3 the model is described, while the results are presented in Section 4. Section 5 concludes.

2 Literature review

Since the first years of 2000s, with the seminal works by Caillaud & Jullien (2001, 2003), Rochet & Tirole (2003), Gabszewicz & Wauthy (2004), and Armstrong (2006), multi-sided platforms or two-sided markets have become a topic of interest for the economic scholars. More recently, the studies on two-sided markets have addressed several open issues of this matter such as the structure of two-sided markets, the role of information and the equilibrium selection in platforms. In this Section, we present a brief overview of the main strands in the economics of multi-sided platforms, and present our contribution to this literature focusing in particular on studies more closely related with our work.

Most of the studies in the strand of platform markets structure and competition covers the interaction among sellers and buyers through one or more platforms, in a context of perfect and imperfect information. As single-homing and multi-homing behaviors are considered, that is, the possibility for a buyer to choose either only one or more platforms, monopolistic platforms

\footnote{A survey of businesses in these industries is in Evans (2003). For a tentative survey of the literature, see Roson (2005). Rochet & Tirole (2006) presented a roadmap to the burgeoning literature, while for a first review, see Rysman (2009). Recently, Cabral et al. (2019) edited a special issue on platforms of the Journal of Economics & Management Strategy that provides some perspectives on economics research on two-sided markets.}
and competition between platforms are studied in several frameworks of network externalities, and intra- and cross-group external effects.\textsuperscript{2} Caillaud & Jullien (2001) examined a first price competition model between two intermediaries in two-sided market of Internet, and Caillaud & Jullien (2003) analyzed a model of imperfect price competition between intermediation service providers.\textsuperscript{3} Rochet & Tirole (2003) built a model of price competition between platforms with two-sided markets and compared the outcomes with those under an integrated monopolist and a Ramsey planner. Gabszewicz & Wauthy (2004) modeled a duopoly competition between two platforms, when agents are heterogeneous on both sides of the market and are allowed to multi-home. Armstrong (2006) presented a model of monopoly platform and a model of competing platforms where agents join a single platform, and a model of “competitive bottlenecks” where one group of agents joins all platforms. More recently, Ambrus & Argenziano (2009) investigated pricing decisions in two-sided markets with network externalities, where consumers are heterogeneous in how much they value the externality. Weyl (2010) developed a general theory of monopoly pricing of networks proposing a model of heterogeneity in which users differ in their income or scale.\textsuperscript{4}

On another strand of literature, several scholars investigated the effect of imperfect information in two-sided markets competition. Halaburda (2013) assumed that both sides have uncertainty about the exchanged quality ex-ante, and that they acquire this information when they choose a certain platform, so the ex-post information is private; they demonstrated that competition, in this framework, could lead to a market failure, but multi-homing could solve this failure. Hagiu & Jullien (2014) assumed instead that consumers have imperfect information and passive expectations while producers have full information about the goods, finding that the level of information detained by the consumers affects positively the level of price competition in case of more than one platform, while in case of monopoly the higher the information detained by the consumers the higher the profits the platform gains. Finally, Jullien & Pavan (2018) studied how the dispersion of information about users’ preferences affects demands and prices through the introduction of uncertainty. Besides Halaburda (2013) and Jullien & Pavan (2018)

\textsuperscript{2}For models of two-sided markets with intra-group and cross-group externalities, see for example Belleflamme & Toulemonde (2009), Belleflamme & Peitz (2010), Casadesus-Masanell & Halaburda (2011), Belleflamme & Toulemonde (2016), and Gabszewicz & Wauthy (2014).

\textsuperscript{3}Damiano & Li (2008), following the idea of Caillaud & Jullien (2003), studied the price competition when matchmakers use prices to sort heterogeneous participants into competing matching markets.

\textsuperscript{4}White & Weyl (2016) extended this theory to a duopoly model.
that have a discrete timing, most of these models are static. Following a dynamic approach, Grossmann et al. (2016) analyzed the dynamic competition between two platform in two-sided markets with network externalities over two periods. In a first stage, a platform wins the contest and serves the two-sided market monopolistically in a second stage. They showed that a head start of one platform does not guarantee future success and the combination of cost advantages and network externalities affects the platforms’ success. In Halaburda et al. (2016) the platform that dominated the market in the last period becomes “focal” in the current period, and hence the buyers have a preference for it. Since consumers have information only on the focal firm and not on the new ones, and they have no information about the quality, the authors showed that, depending on the considered time horizon, a low-quality platform could prevail in the market if it is focal.

Another strand of literature introduced product differentiation in multi-sided platforms competition. The first framework for analyzing two-sided markets with different degrees of product differentiation on each side of the market is provided by Armstrong & Wright (2007), who showed that “competitive bottlenecks” arise endogenously. Hagiu (2009) provided a model to analyze the platforms competition in two-sided markets where indirect network effects are determined endogenously by consumers’ taste for variety. Reisinger (2012) analyzed a two-sided market model with differentiated platforms which compete for advertisers and users. Gabszewicz & Wauthy (2014) modeled platform competition in a two-sided market with cross network externalities when the preferences of individuals are heterogeneous in their valuation of these externalities, and they showed that platform competition implies a vertical differentiation both when platforms commit to prices and when they commit to network sizes. Lee (2014) examined the case when a market will sustain a single or multiple platforms by applying a model of a bilateral contracting among platforms with externalities across both contracting and non-contracting partners to a marketplace competition, where two competing marketplaces compete for differentiated product retailers to join their respective sites. Ribeiro et al. (2016), merging the two-sided markets duopoly model of Armstrong (2006) with the differentiation model of Gabszewicz & Wauthy (2012), showed that in equilibrium, the high-quality platform sells at

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5Peitz et al. (2017) studied an optimal experimentation by using a Bayesian learning framework to test the dynamic pricing problem of a monopolistic platform in a two-sided market, where the platform provider is uncertain about the strength of the externality.

6For examples of network externalities in vertical differentiation models, see Argenziano (2008), Baake & Boom (2001), Griva & Vettas (2011), Navarro (2012), and Garcia & Vergari (2016).
a higher price and captures a greater market share than the low-quality platform, despite the indifferent consumer being closer to the high-quality platform. Roger (2017) studied a duopoly in which two-sided platforms compete in differentiated products in a two-sided market with cross-market externality and single-homing behavior. Veiga et al. (2017) proposed a model of platforms which have an incentive to design non-price features to attract valuable users, in a monopolistic one-sided platform/network by combining the of model Weyl (2010) with the one of Veiga & Weyl (2016). Zennyo (2016) investigated the competition between vertically differentiated platforms in two-sided markets where the two competing platforms can produce either higher- or lower-quality devices for consumers.7

Studies more in line with our work can be found within the literature on platform investment strategies. In particular, Anderson Jr et al. (2013) investigated as a key decision in platform design the level of platform performance to invest in new product development such as the choice between investing in platform performance or holding back investment to facilitate third party content development in markets that exhibit two-sided network externalities. Hagiu & Spulber (2013) introduced investment in first-party content as a strategic variable for two-sided platforms and show the interplay with the platforms pricing strategies to solve market the coordination problem, when two-sided platforms provide first-party content which makes participation more attractive to buyers, independently of the presence of the sellers. Belleflamme & Peitz (2019) studied the platform non-price strategies such as the product visibility and quality control when platforms have an interest to guide consumers to products they like and when they may control the quality of sellers and remove underperforming sellers from the platform. Our paper contributes to the extant literature on platform investment strategies by looking at the interaction between those and seller competition. This is done not only by looking at how investment incentives are affected the degree of competition, but also analyzing how investment opportunities for the platform can affect the sign of the within-group external effects.

3 The model

The model considers a population of $n_s$ sellers and $n_b$ buyers interacting on a monopolistic platform. Sellers are competitors offering horizontally differentiated products. Production is

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7Serio et al. (2016) developed a model that exploits the two-sided market of flights of different quality.
characterized by constant returns to scale, so that the total cost function for seller $k$ producing a quantity $q_k$ is given by $C(q_k) = q_k$. Buyers’ utility function is given by:

$$U(q_0; q_1, q_2, ..., q_{n_s}) = A \sum_{k=1}^{n_s} q_k - \frac{1}{2} \left( \sum_{k=1}^{n_s} q_k^2 + \lambda \sum_{k=1}^{n_s} \sum_{g \neq k} q_k q_g \right) + q_0$$  \hspace{1cm} (1)$$

$q_0$ is an Hicksian composite commodity with price $p_0 = 1$. Parameter $\lambda$ (with $0 < \lambda \leq 1$), measures the degree of substitutability between products (for $\lambda = 1$, for products are homogeneous). $A$ is a parameter affecting symmetrically the marginal utility of products offered by sellers. We shall assume $A = 1 + \bar{A} + a$, where $\bar{A} > 0$ and $a \geq 0$ is the level of investment in demand-enhancement by the platform, whose associated cost is $C_A(a) = \frac{\gamma a^2}{2}$, with $\gamma > 0$. The higher $\gamma$, the less appealing will be the investment.

The buyer maximizes her utility $U(q_0; q_1, q_2, ..., q_{n_s})$ subject to the budget constraint $y = q_0 + \sum p_k q_k$, where $y$ is the income and $p_k$ is the price of $q_k$. This yields the linear demand function, where $q_{-k} = \sum_{g \neq k} q_g$:

$$p_k = A - q_k - \lambda q_{-k}$$

The net surplus of visiting the platform for sellers and buyers is given by $v_s = r_s + \pi(n_b, n_s) - m_s$ and $v_b = r_b + u(n_b, n_s) - m_b$. $r_g$ is the stand-alone utility on side $g \in \{b, s\}$, $\pi(n_b, n_s)$ and $u(n_b, n_s)$ are the net gains for trade for sellers and buyers respectively, while $m_g$ is the subscription fee imposed by the platform on side $g$.

In the current version of the paper, we shall assume that the number of buyers is exogenously given and equal to 1 and that $n_s > 1$. However, the platform can affect $\pi(n_b, n_s)$ and $u(n_b, n_s)$ in two ways: i) by determining $A$ and ii) by levying a transaction fee $f_s$ to sellers (while $f_b = 0$ by assumption). It follows that the net gains for trade for sellers is given by market profit net of the total transaction fees paid to the platform. As for the nature of competition, we will consider both the Cournot and Bertrand cases. Finally, the timing of the game is such that, at stage $t = 0$, the platform fixes $a$ and $f_s$, while at stage $t = 1$, after having observed $a$ and $f_s$, sellers compete fixing quantities (or prices) simultaneously.
3.1 Cournot competition

3.1.1 Stage t=1: sellers’ choice

The (net) profit function for seller $k$ is:

$$\pi_k = (A - q_k - \lambda q_k) q_k - (1 + f_s)q_k$$  \hspace{1cm} (2)

The derivation of equilibrium for the Cournot case is standard, and yields the following values:

$$q^*_k = \frac{A - 1 - f_s}{2 + \lambda(n_s - 1)}$$ \hspace{1cm} (3a)

$$p^*_k = \frac{A - 1 - f_s}{2 + \lambda(n_s - 1)} + 1 + f_s$$ \hspace{1cm} (3b)

$$Q^* = n_s \frac{A - 1 - f_s}{2 + \lambda(n_s - 1)}$$ \hspace{1cm} (3c)

3.1.2 Stage t=0: platform choice

By plugging (3c) into the platform profit function, we obtain:

$$\Pi_P = m_s n_s + m_b + f_s Q^* - \frac{\gamma a^2}{2} = m_s n_s + m_b + f_s n_s \frac{A - 1 - f_s}{2 + \lambda(n_s - 1)} - \frac{\gamma a^2}{2}$$  \hspace{1cm} (4)

with $A = 1 + \bar{A} + a$. Solving the maximization problem for the platform, we have the equilibrium levels for $a$ and $f_s$:

$$a^* = \bar{A} \frac{n_s}{2\gamma (2 + \lambda(n_s - 1)) - n_s}$$ \hspace{1cm} (5)

$$f^*_s = \bar{A} \frac{\gamma (2 + \lambda(n_s - 1))}{2\gamma (2 + \lambda(n_s - 1)) - n_s}$$ \hspace{1cm} (6)

The second order conditions for a maximum is:

$$\gamma > \frac{n_s}{2(2 + \lambda(n_s - 1))} \equiv \gamma^{SOC}$$ \hspace{1cm} (7)

Hereafter, we assume that $\gamma > \gamma^{SOC}$. 
Plugging (5) and (6) into (2) and (4), we obtain:

\[ \Pi^*_P = \frac{A^2 \gamma n_s}{4 \gamma [2 + \lambda(n_s - 1)] - 2n_s} + m_sn_s + mb \]  

\[ \pi^*_k = \frac{A^2 \gamma^2}{[2\gamma(2 + \lambda(n_s - 1)) - n_s]^2} \]

3.2 Bertrand competition

3.2.1 Stage t=1: sellers’ choice

We now analyse how the levels of optimal investment and fees change if the competition among firms is à la Bertrand. The competition among the firms leads to the following equilibrium price and quantity (the same for all the firms):

\[ p^*_k = \frac{2 + a + \bar{A} + fs}{2} \]  

\[ q^*_k = \frac{a + \bar{A} - fs}{2[1 + \lambda(n_s - 1)]} \]  

\[ Q^* = n_s \frac{a + \bar{A} - fs}{2[1 + \lambda(n_s - 1)]} \]

The profit of each firm will be equal to:

\[ \pi^*_k = \frac{(a + \bar{A} - fs)^2}{4[1 + \lambda(n_s - 1)]} \]

3.2.2 Stage t=0: platform choice

Plugging (10c) into the platform profit function we obtain:

\[ \Pi_P = m_sn_s + mb + n_sf_s \left(\frac{a + \bar{A} - fs}{2[1 + \lambda(n_s - 1)]}\right) - \gamma \frac{a^2}{2} \]

Maximizing (12) with respect to \( a \) and \( fs \) yields:

\[ a^* = \frac{\bar{A}n_s}{4\gamma[1 + \lambda(n_s - 1)] - n_s} \]  

\[ fs^* = \frac{2\bar{A}\gamma(1 + \lambda(n_s - 1))}{4\gamma[1 + \lambda(n_s - 1)] - n_s} \]
The SOCs are verified if:

\[ \gamma > \frac{n_s}{4[1 + \lambda(n_s - 1)]} \equiv \gamma_{SOC}^{S} \]  

(15)

Plugging (13) and (14) into (11) and (12), we obtain:

\[ \Pi_p = m_s n_s + m_b + \frac{\bar{A}^2 \gamma n_s}{2[4 \gamma (1 + \lambda(n_s - 1)) - n_s]} \]  

(16)

\[ \pi_k^* = \frac{9 \bar{A}^2 \gamma^2 (1 + \lambda(n_s - 1))}{[4 \gamma (\lambda(n_s - 1) + 1) - n_s]^2} \]  

(17)

4 Results

This Section derive the main results of the paper, showing in particular how investment, fees and profits are affected by the intensity of competition among sellers, measured by the degree of substitutability, number of sellers and the form of competition (Cournot vs Bertrand).

4.1 Cournot competition

4.1.1 The determinant of investment and transaction fees

The first set of results concern investment in quality improvement and transaction fees, and how these variables, chosen by the platform, are affected by \( \lambda \), \( n_s \), \( \bar{A} \), and \( \gamma \). Straightforward derivations show that:

\[ \frac{\partial a^*}{\partial \lambda} = -\frac{2 \bar{A} \gamma n_s (n_s - 1)}{[2 \gamma (2 + \lambda(n_s - 1)) - n_s]^2} < 0 \]

\[ \frac{\partial f_k^*}{\partial \lambda} = -\frac{\bar{A} \gamma n_s (n_s - 1)}{[2 \gamma (2 + \lambda(n_s - 1)) - n_s]^2} < 0 \]

that is, more intense competition between sellers leads the platform to invest less and to fix lower fees. When competition is intense, the value of platform investment is mostly appropriated by consumers, which impede the platform to take advantage of it through higher fees. As a consequence, investment is reduced. As for the effect of \( n_s \) we obtain:

\[ \frac{\partial a^*}{\partial n_s} = \frac{2 \bar{A} \gamma (2 - \lambda)}{[2 \gamma (2 + \lambda(n_s - 1)) - n_s]^2} > 0 \]

\[ \text{Notice that } \gamma_{SOC}^{C} > \gamma_{SOC}^{B} \text{ if } \lambda > 0, \text{ and we assumed that } \gamma > \gamma_{SOC}^{C}. \]
\[
\frac{\partial f^*_s}{\partial n_s} = \frac{\bar{A}\gamma(2 - \lambda)}{[2\gamma (2 + \lambda(n_s - 1)) - n_s]^2} > 0
\]

The larger is the number of sellers, the larger is total quantity sold in the market. This increases the return to investment for the platform, which as consequence increases the fees as well. Finally,

\[
\frac{\partial a^*}{\partial \bar{A}} = \frac{n_s}{2\gamma (2 + \lambda(n_s - 1)) - n_s} > 0
\]

\[
\frac{\partial f^*_s}{\partial \bar{A}} = \frac{\gamma(2 + \lambda(n_s - 1))}{2\gamma (2 + \lambda(n_s - 1)) - n_s} > 0
\]

\[
\frac{\partial a^*}{\partial \gamma} = -\frac{2\bar{A}n_s(2 + \lambda(n_s - 1))}{[2\gamma (2 + \lambda(n_s - 1)) - n_s]^2} < 0
\]

\[
\frac{\partial f^*_s}{\partial \gamma} = -\frac{\bar{A}n_s(2 + \lambda(n_s - 1))}{[2\gamma (2 + \lambda(n_s - 1)) - n_s]^2} < 0
\]

Straightforwardly, an increase in \( \bar{A} \) (decrease in \( \gamma \)) increases the return (decreases the cost) of investment, leading to a higher value of \( a \). As a consequence, the platform increases \( f_s \) to appropriate the increase in sellers’ profit.

### 4.1.2 The sign of sellers’ within-group externalities and platform profit

We now turn to sellers’ and platform profit. As the main of contributions of this paper, we first look at the impact of parameters on sellers’ profit, in particular to the impact of varying the number of sellers on their individual profit. This is shown in the next Proposition.

**Proposition 1** \( \pi^*_k \) is strictly increasing in \( n_s \) when \( \gamma < \frac{1}{2\lambda} \equiv \gamma_C \). Moreover, \( \pi^*_k \) is strictly decreasing in \( \gamma \) and in \( \lambda \), and increasing in \( \bar{A} \).

**Proof.** The partial derivative of \( \pi^*_k \) with respect to \( n_s \) is equal to \( \frac{2\bar{A}^2\gamma^2(1-2\gamma \lambda)}{[2\gamma (2+\lambda(n_s-1)) - n_s]^2} \). Notice that the denominator is positive when \( \gamma > \gamma_{SOC}^C \), which we assumed to hold, and the numerator is positive when \( \gamma < \frac{1}{2\lambda} \). Hence, \( \frac{\partial \pi^*_k}{\partial n_s} > 0 \) if \( \gamma < \frac{1}{2\lambda} \).

The sign of \( \frac{\partial \pi^*_k}{\partial \gamma} = \frac{2\bar{A}^2\gamma n_s}{[2\gamma (2+\lambda(n_s-1)) - n_s]^2} \) is negative, since the derivative shares the same denominator of \( \frac{\partial \pi^*_k}{\partial n_s} \), and the numerator is always positive.
\[
\frac{\partial \pi^*_k}{\partial \lambda} = -\frac{4A^2\gamma^3(n_s-1)}{[2\gamma(2+\lambda(n_s-1))-n_s]^3}
\] is negative since both its numerator and its denominator are positive.

Finally, \[
\frac{\partial \pi^*_k}{\partial \bar{A}} = \frac{2A^2\gamma^2}{[2\gamma(2+\lambda(n_s-1))-n_s]^2}
\] is positive since its numerator and its denominator are always positive.

This completes the proof. ■

The intuition for the second part of the Proposition is straightforward. A reduction in \(\gamma\) has an impact on sellers’ profit through through \(a\) and \(f_s\), with the former being more sensitive than the latter. As consequence, seller’s profit increases when the demand-enhancing technology becomes more effective. Variations of \(\lambda\) and \(\bar{A}\) have both a direct impact on profit and an indirect effect through \(a\) and \(f_s\). However, the two effects go in the same direction. In particular, an increase in product substitutability has a negative impact on seller’s profit because makes market competition more intense and because it reduces investment by the platform.

When we consider instead the impact of \(n_s\) on \(\pi^*_k\), two opposing forces are at work. On one hand, an increase in \(n_s\) increases competition, so reducing sellers’ profit. This the standard negative within-group external effect, present in Belleflamme & Peitz (2019). However, a larger number of sellers makes the investment by the platform more profitable. It turns out that when the demand-enhancing technology is sufficiently productive \((\gamma < \bar{\gamma}_C)\), the second effect prevails, and the within-group external effects affecting sellers turn positive. \(\bar{\gamma}_C\) is decreasing in \(\lambda\): when product substitutability is low, sellers’ profit are less affected by variations in \(n_s\). Moreover, we observe that \(\bar{\gamma}_C > \gamma^{SOC}_C\) for any value of \(n_s\), i.e. the positive external effect can be always be observed. Nevertheless, \(\gamma^{SOC}_C\) tends to \(\bar{\gamma}_C\) as \(n_s\) increases, since \[
\lim_{n_s \to \infty} \frac{n_s}{\gamma[2\gamma(2+\lambda(n_s-1))-n_s]} = \frac{1}{2\lambda}.
\]

How platform profit is affected by the parameters of the model is summarized in Proposition 2.

**Proposition 2** \(\Pi^*_P\) is strictly increasing in \(n_s\) and in \(\bar{A}\), and strictly decreasing in \(\gamma\) and in \(\lambda\).

**Proof.** \(\Pi^*_P\) is increasing in \(n_s\) since \[
\frac{\partial \Pi^*_P}{\partial n_s} = m_s + \frac{\bar{A}^2\gamma^2(2-\lambda)}{[2\gamma(2+\lambda(n_s-1))-n_s]^2}
\] is positive, given that \(m_s > 0\) and the fraction has both positive numerator and positive denominator.

The same is true for \[
\frac{\partial \Pi^*_P}{\partial \bar{A}} = \frac{\bar{A}n_s}{2\gamma(2+\lambda(n_s-1))-n_s}
\] as long as \(\gamma > \gamma^{SOC}_C\).

The derivative of \(\Pi^*_P\) with respect to \(\gamma\) is equal to \(-\frac{\bar{A}^2n_s^2}{2[2\gamma(2+\lambda(n_s-1))-n_s]^2}\) and is always negative.

Finally, \[
\frac{\partial \Pi^*_P}{\partial \lambda} = -\frac{\bar{A}^2\gamma^2(n_s-1)n_s}{[2\gamma(2+\lambda(n_s-1))-n_s]^2}
\] is negative when \(n_s > 1\).

This completes the proof. ■

Proposition 2 shows that platform profit in equilibrium moves together with the investment in demand-enhancement. Obviously, the platform invests more when there are the conditions to
make in the investment more profitable. As the for the impact on $n_s$ in particular, we observe that a larger number of sellers has a positive impact on profits because it increases both the transaction fees and because and the volume of sales.

4.2 Bertrand competition

4.2.1 The determinant of investment and transaction fees

We now study how investment in quality improvement and transaction fees are influenced by changes in $\lambda$, $n_s$, $\bar{A}$, and $\gamma$ in the case of Bertrand competition. For what concerns the degree of substitutability between products, we have that:

$$\frac{\partial a^*}{\partial \lambda} = -\frac{4\bar{A}\gamma(n_s - 1)}{[4\gamma(1 + \lambda(n_s - 1)) - n_s]^2} < 0$$

$$\frac{\partial f_s^*}{\partial \lambda} = -\frac{2\bar{A}\gamma(n_s - 1)}{[4\gamma(1 + \lambda(n_s - 1)) - n_s]^2} < 0$$

As the intensity of competition between sellers arises, the platform invests less and fixes lower transaction fees. As in the Cournot case, when the competition is intense, the value of the platform investment is mainly enjoyed by consumers, who do not pay any fee.

When the number of sellers increases, we obtain:

$$\frac{\partial a^*}{\partial n_s} = -\frac{4\bar{A}\gamma(\lambda - 1)}{[4\gamma(1 + \lambda(n_s - 1)) - n_s]^2} > 0$$

$$\frac{\partial f_s^*}{\partial n_s} = -\frac{2\bar{A}\gamma(\lambda - 1)}{[4\gamma(1 + \lambda(n_s - 1)) - n_s]^2} > 0$$

If the number of sellers increases, the quantity sold in the market increases as well, leading to a higher effect of the quality-improving investment; as a consequence, higher fees can be requested to sellers.

Concerning the other parameters, we have:

$$\frac{\partial a^*}{\partial \bar{A}} = \frac{n_s}{4\gamma(1 + \lambda(n_s - 1)) - n_s} > 0$$
As one could expect, an increase in $\bar{A}$ (decrease in $\gamma$) increases the return (decreases the cost) of investment, leading to a higher value of $a$. As a consequence, the platform increases $f_s$ to appropriate the increase in sellers’ profit.

4.2.2 The sign of sellers’ within-group externalities and platform profit

The nature of competition (Cournot vs Bertrand) does not affect how sellers’ and platform profit are affected by the parameters of the model. Proposition 3 and Proposition 4 follow, which share the same intuition of Proposition 1 and 2 respectively.

**Proposition 3** The seller’s profit is strictly decreasing in $\lambda$ and $\gamma$, strictly increasing in $\bar{A}$, and strictly increasing in $n_s$ if $\gamma < \frac{2+\lambda(n_s-2)}{4\lambda(1+\lambda(n_s-1))} \equiv \bar{\gamma}_B$ (with $\bar{\gamma}_B > \gamma_{SOC}$).

**Proof.**
\[
\frac{\partial \pi^*_k}{\partial \lambda} = -\frac{9\bar{A}^2\gamma^2(n_s-1)(4\gamma(1+\lambda(n_s-1))+n_s)}{(4\gamma(1+\lambda(n_s-1))-n_s)^3}
\]
is negative, since the numerator is positive when $n_s > 1$ and the denominator is positive when $\gamma > \gamma_{SOC}$.

The derivative of the seller $k$’s profit with respect to $\gamma$, \[
\frac{\partial \pi^*_k}{\partial \gamma} = -\frac{18\bar{A}^2\gamma n_s(1+\lambda(n_s-1))}{(4\gamma(1+\lambda(n_s-1))-n_s)^2},
\]
is negative since both the numerator and the denominator of the fraction are positive.

The sign of \[
\frac{\partial \pi^*_k}{\partial A} = \frac{18\bar{A}^2(1+\lambda(n_s-1))}{(4\gamma(1+\lambda(n_s-1))-n_s)^2}
\]
is positive since both the numerator and the denominator are positive.

Finally, the partial derivative of $\pi^*_k$ with respect to $n_s$ is equal to \[
\frac{9\bar{A}^2\gamma^2(2-\lambda(4\gamma(1+\lambda(n_s-1))-n_s+2))}{(4\gamma(1+\lambda(n_s-1))-n_s)^3}
\]
has a positive denominator (as long as $\gamma > \gamma_{SOC}$), and a positive numerator if $\gamma < \frac{2+\lambda(n_s-2)}{4\lambda(1+\lambda(n_s-1))}$, hence when $\gamma < \bar{\gamma}_B$ this derivative is positive.

This completes the proof.

**Proposition 4** $\Pi^*_p$ is strictly increasing in $n_s$ and in $\bar{A}$, and strictly decreasing in $\gamma$ and in $\lambda$.
Proof. The platform profit is increasing in the number of sellers, that is, \( \frac{\partial \Pi^*_P}{\partial n_s} = m_s + \frac{2A^2\gamma^2(1-\lambda)}{(4\gamma(1+\lambda(n_s-1))-n_s)^2} > 0 \), since \( m_s > 0 \) and both the numerator and the denominator of the fraction are positive.

\( \frac{\partial \Pi^*_P}{\partial \bar{A}} = \frac{\bar{A}\gamma n_s}{4\gamma(1+\lambda(n_s-1))-n_s} \) is positive since its numerator and its denominator are positive.

The derivative of \( \Pi^*_P \) is decreasing in \( \gamma \) since both the numerator of the denominator of \( \frac{\partial \Pi^*_P}{\partial \gamma} = -\frac{A^2n_s^2}{2(4\gamma(1+\lambda(n_s-1))-n_s)^2} \) are positive.

Finally, \( \frac{\partial \Pi^*_P}{\partial \lambda} = -\frac{2\lambda^2n_s^2}{(4\gamma(1+\lambda(n_s-1))-n_s)^2} \) is negative since both the numerator and the denominator of the fraction are positive.

This completes the proof. ■

4.3 A comparison between Cournot and Bertrand

As a final result, we compare the outcomes we obtain under Cournot and Bertrand competition. By comparing the corresponding equilibrium values for \( a \) and \( f_s \), it turns out that investment and fees are higher under Cournot competition, as a result of the lower intensity of competition in this case, which increases the marginal return of \( a \) for sellers, and then the possibility for the platform to appropriate the additional profit through higher fees. Moreover, we can compare the magnitude of the within-group external effect by comparing \( \bar{\gamma}_C \) and \( \bar{\gamma}_B \). It turns out that \( \bar{\gamma}_C > \bar{\gamma}_B \left( \frac{1}{2\lambda} > \frac{2+\lambda(n_s-2)}{4\lambda(1+\lambda(n_s-1))} \right) \) for all \( \lambda > 0 \). That is, the seller within-group external effect (with its own sign) is always greater in Cournot than in Bertrand, or, told differently, the external effect is positive for a larger area of the parameter space under Cournot competition.

The intuition is that this is the sum of two effects. On one hand, under Cournot the intensity of market competition is lower, and so it is the direct impact on profit of an increasing number of sellers. On the other hand, going in the same direction, the platform incentives to invest are higher under Cournot.

5 Conclusions

In this paper we develop a model where a monopolistic platform can invest in quality improvement, which affect buyers’ utility and in this way sellers’ profits. We looked at how platform incentives interact with seller’s competition in determining seller’s and platform profit. We found that, irrespectively of form of competition (Cournot vs Bertrand), the possibility for the
platform to invest in quality can turn the negative within-group external effects due to seller competition into a positive effect. This has, of course, important consequences in platform pricing strategies to attract buyers and sellers into the platform. Making the number of buyers and sellers endogenous is the obvious next step for our analysis. Moreover, it seems interesting to look at competition between platforms as well, to see how it can affect investment in platform quality, the sign of external effects and in turn the pricing strategies of platforms to attract users on the two sides of the market.

References


