Trade Liberalization, Selection and Technology Adoption with Vertical Linkages.*

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Abstract

This paper analyses the role played by vertical linkages on the effects of trade liberalization on technology adoption and their consequences on average productivity and welfare in a trade model with heterogeneous firms. We find that the strength of vertical linkages shapes the effects that trade liberalization produces on firms' survival and technology upgrading decisions, having an impact on the average productivity of the economy and, ultimately, on welfare.

Keywords: trade liberalization, heterogeneity, selection, technology adoption, vertical linkages.

J.E.L. Classification: F1.

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1 Introduction

The last two decades have witnessed the birth of a rich literature that theoretically and empirically shows that firm heterogeneity is the key to understanding the impact of trade liberalization policies on an industry's average productivity. According to this literature, trade increases average productivity due to a reallocation effect that operates through a selection mechanism (trade expels the less efficient firms out of the market and reallocates production factors towards the most efficient units (Melitz, (2003)) and through a within plant effect. Both sources, selection and plant productivity growth, have been found to be empirically relevant when examining the impact of trade liberalization on an industry's average productivity (Pavnick (2002), Trefler (2004), Bloom, Draca and Van Reenen (2016)).

While the first branch of this literature has focused exclusively on firm's heterogeneity in export activities, a very recent avenue points out that this dimension is far more complex with industries heavily relying on vertical linkages and the use of intermediate inputs. In this literature importing intermediate inputs also plays an important role in determining the ultimate effect of trade on an industry's average productivity (Kasahara and Rodrigue (2008), Halpern, Koren and Szeidl (2015)) Whilst this literature documents a prominent role of intermediate inputs in plant productivity growth, recent empirical evidence suggests that, following trade liberalization firms also increase their productivity by rising their investment in R&D activities (Bustos (2011), Lileeva and Trefler (2010), Aw, Robers and Xu (2011)).[1] The current paper contributes to the literature by examining theoretically the interaction between these two sources of plant productivity growth, that is, importing intermediates (vertical linkages) and technology upgrading, a relatively unexplored mechanism in this literature. More precisely, this work investigates the role played by the presence of vertical linkages among firms that use both locally produced and imported intermediates on the impact of trade liberalization policies on technology upgrading, average productivity and, ultimately, welfare.

According to the OECD, vertical linkages are defined as links between firms up and down in the production chain. Theoretically, vertical linkages are modelled as the use of final output produced by other firms as intermediate inputs in the production process. The existence of these links through intermediates implies that firms are interconnected in such a way that a shock that affects the production of one of these goods is likely to be transmitted to the production of other final goods thereby generating effects that go beyond the original ones. In the case in which firms use as intermediate inputs a share of the final production of other firms within the same industry, this creates a within-industry multiplier effect that could, in principle, increase the firms' profits to technology upgrade. Yet, this comes with a double side: the multiplier effect could intensify firms' competition for scarce production factors in such a way that their prices would be driven up, reducing operating profits and, eventually, deterring firms from adopting more productive technologies.

Our paper is motivated by the fact that vertical linkages not only bring in new channels that could modify the link between trade liberalization, productivity and welfare, but also by the recognition that they are empirically relevant.

[1] See also the seminal work by Atkeson and Burstein (2010) and a more recent contribution by Impulliti and Licandro (2016).

Indeed, intermediate inputs constitute a fundamental part of the production structure of an economy. Recent studies find that intermediate inputs accounts for around 50 to 60% of total gross output.[2] Intermediates are also responsible for a large share of the total volume of international trade with the World Trade Report 2014 (p. 43) stating that "the average import content of exports is around 25 per cent – and increasing over time – and almost 30 per cent of merchandise trade is now in intermediate goods or components". Moreover, Di Giovanni and Levchenko (2010) use input-output tables from the UNIDO Industrial Statistics Database for a panel of 28 manufacturing industries over 55 developed and developing countries and they find that the largest part of the volume of intermediate inputs consumed by a manufacturing industry comes from inputs produced by the same industry, although sectors differ substantially in the extent to which they use them. These findings not only reinforce the idea that industry-specific vertical linkages are relevant at the sectoral level, but also suggest that differences in the strength of vertical linkages (the intensity in the use of intermediate inputs) can result in differences in export behavior or technology upgrading across industries.

To analyze the role played by vertical linkages in determining the effects of trade liberalization on technology upgrading, productivity and welfare, we construct a trade model with heterogeneous firms that are interconnected by vertical linkages in such a way that all firms can employ, as intermediate inputs, the final goods produced by both domestic and foreign exporting firms, as in Krugman and Venables (1995) and Nocco (2012).[3] These firms are also allowed to upgrade the current state of the technology they use by reducing their marginal cost to be a fixed proportion γ of its initial value, bearing a fixed cost of adoption of f_I units of the final good, as in Bustos (2011) and Navas and Sala (2015) among others. While simple, this framework is consistent with the stylized fact that many firms within an industry do not engage in R&D activities (Klette and Kortum (2004)).

Our results suggest that the strength of vertical linkages shapes the effects that trade liberalization produces on firms' survival and technology upgrading decisions, having an impact on the average productivity of the economy and, ultimately, on welfare. The model exhibits different types of equilibria depending on the parameter configuration, which are related to different hierarchies among adopters, exporters and domestic firms. Specifically, if *technology adoption* is an expensive activity relative to exporting, the most productive firms engage in both technology adoption and exporting, the less productive ones serve the domestic market using their original technology and the firms in the middle rank export using their original technology.[4] If, instead, *exporting* is relatively more expensive, the firms in the middle rank adopt the new technology but do not export.[5] Finally, for intermediate trade costs, an equilibrium in

[2] See, for instance, Jones (2011, 2013).

[3] Baldwin et al. (2005, chp 8) stresses the importance of this type of modelling vertical linkages (input-output linkages). Their graduate textbook on New Economic Geography suggests that: "[...] Perhaps the most important is that of the so-called "vertical linkage" models (VL models for short). In these models, input-output linkages[...]. The seminal paper, Venables (1996a), introduces cost linkages between an upstream sector and a downstream sector. For the sake of simplicity, Krugman and Venables (1995) and Fujita Krugman and Venables (1999 chapter 14) collapse the two sectors into one, so input-output relationships switch from 'vertical' linkages to 'horizontal' linkages (nevertheless the VL label is retained)."

[4] This equilibrium, analysed in depth by Bustos (2011), is consistent with the empirical evidence provided in the same paper.

[5] Castellani and Zanfei (2007) find that a non-negligible proportion of Italian manufacturing firms are domestic innovators. This equilibrium is consistent with this evidence.

which firms are adopters and exporters or domestic firms relying on the original technology may exist.[6] The effects of trade liberalization on technology upgrading will vary with the intensity of vertical linkages and across equilibria (when the intensity is either low or high). For intermediate levels, however, the results are common across equilibria: trade liberalization promotes technology upgrading when the intensity is intermediate-low and deters technology upgrading when it is intermediate-high.

These results on technology adoption and productivity have a clear impact on the effects of trade liberalization on welfare. Specifically, trade liberalization will have a positive impact on welfare when the intermediate input intensity is low or intermediate-low but it will have a negative impact when it is intermediate-high and high. We also find that both channels, vertical linkages and technology upgrading magnify the effects of trade liberalization on welfare through its impact on the price of intermediate goods. To get a flavour of the quantitative importance of these channels, in the last part of the paper a simulated version of the model for different degrees of vertical linkages is presented. Although the results should be interpreted as illustrative, the simulations suggest that these magnification effects on welfare are also quantitatively important. Compared to a version of the model without vertical linkages, the effects of a reduction in trade costs from 75% to 15% on welfare in our benchmark model almost double.[7] In addition, the possibility of firms to technology upgrade contributes to an increase in the impact of a reduction in trade costs from 75% to 15% on welfare of approximately 13%.

This paper is related to different strands of the literature. The very first one is the emerging literature on macroeconomics that explores the consequences of vertical linkages for explaining cross-country income and productivity differences (Ciccone (2002), Jones (2011), Klenow and Rodriguez-Clare (2005)) or the more recent works of Fadinger, Ghiglino and Teteryatnikova (2017) and Cunat and Zymek (2017), which include vertical linkages in a multi-sector model of trade and heterogeneous firms. However these models abstract from an important channel through which vertical linkages may affect productivity differences: technology adoption. Vertical linkages have been also used in macroeconomics to investigate business cycle transmission across countries (Di Giovanni and Levchenko (2010)).[8]

The second one is the literature on trade which emphasizes the importance of vertical linkages for aggregate trade flows and the impact of trade liberalization on welfare (Helpman and Krugman (1987), Hummels, Ishii and Yi (2001), Nocco (2012), Costinot and Rodriguez-Clare (2014), Ossa (2015), Caliendo, Feenstra, Romalis and Taylor (2015)). This literature outlines how vertical linkages magnify the gains from trade. In these models a reduction in trade costs generates a further reduction in final good prices through its impact in the cost of intermediate inputs. This literature

[6] See Lileeva and Trefler (2010) for evidence supporting this equilibrium.

[7] This is the case when the intensity of vertical linkages is moderate (i.e. 40%). According to the data reported by Jones (2011), this corresponds to a value close to the average vertical intensity observed in a cross-section of countries, included both developing and developed countries.

[8] Moreover, Lee, Padmanabhan and Whang (1997) have analyzed the so called “bullwhip effect” that underlines how small changes in final demand can cause a big change in the demand for intermediate goods along the value chain.

however leaves aside technology upgrading and more precisely the role played by vertical linkages on the firm's decision to technology upgrade, which is the main focus of this paper.

Another recent strand of literature explores the impact of trade on firms' technology upgrading decisions in an environment characterized by heterogeneous firms (Rubini (2014), Perla, Tonetti and Waugh (2014), Navas and Sala (2015)), where intermediate inputs are absent. More closely related to our work is the literature that analyses the role played by trade in intermediates on firms' innovation decisions. Bas and Berthou (2017), Boler, Moxnes and Ullveit-Moe (2015) focus on studying the complementarity between importing and innovation decisions at the firm level.[9] These papers however do not consider the roundabout structure of production and therefore industry-specific linkages are not explored in this context. Fieler et al. (2018) build a quantitative model of trade with firm heterogeneity, quality upgrading and input linkages to assess the impact of the 1991 Colombian's trade liberalization episode on skill intensity and the different channels behind it. Unlike Fieler et al. (2018), we focus on the impact of trade liberalization on average productivity and welfare when vertical linkages and technology upgrading are present. In addition, our paper rather than focussing on a small open economy, is solved for the equilibrium of a two symmetric open economies and, therefore, the framework developed here is more suitable to identifying new effects of trade liberalization on technology adoption and selection when firms producing in developed countries are interconnected by vertical linkages.

The rest of the paper is organized as follows. Section 2 presents the structure of the model and its different equilibria with explicit solutions. Sections 3 and 4, respectively, discuss the effects of trade liberalization on unit input cutoffs and on welfare. Section 5 concludes.

2 The Model

Let us consider a world in which there are two symmetric countries H and F, each populated by L individuals endowed with one unit of time that is dedicated entirely to work. Individuals derive utility from the consumption of two different goods, T and M, according to the following Cobb-Douglas functional form

$$U = \frac{C_T^{1-\mu} C_M^{\mu}}{\mu^{\mu}(1-\mu)^{1-\mu}} \quad 0 < \mu < 1$$

where the parameter μ represents the proportion of expenditure dedicated to the good M. The good T is a homogenous good, produced with a linear technology (i.e., one unit of labour is required to produce one unit of output) under perfect competition and freely traded. In contrast, M is a differentiated good and individuals derive utility from a continuum of varieties (indexed by $j \in \Omega$) according to the following specification

[9] Bas and Berthou (2017) explores the complementarity between technology adoption and importing intermediates while Boler, Moxnes and Ullveit-Moe (2015) studies this complementarity in a structural model of R&D investments.

$$C_M = \left(\int\limits_{j \epsilon \Omega} C_\varepsilon \left(j\right)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 1$ is the constant elasticity of substitution and $C_\varepsilon(j)$ denotes the individual consumption of variety j.

Each variety in the economy is produced by a single firm in a monopolistic competition environment. Unlike Melitz (2003), firms combine labour and a set of differentiated intermediate inputs to produce their variety using a Cobb-Douglas technology. More precisely, the firm producing the $j-th$ variety uses the following technology

$$q\left(j\right) = \frac{1}{a(j)} L(j)^{1-\alpha} M(j)^{\alpha}$$

where $q\left(j\right)$ is the quantity obtained combining $L(j)$ units of labour and $M(j)$ units of the differentiated intermediate inputs, and $a(j)$ represents the unit input requirements of firm j and it is an inverse measure of the productivity level of the firm. Since $\alpha \in (0,1)$ is the intermediate share and determines the importance of intermediate inputs in the production of the final good, it also represents the *strength of vertical linkages*.

Following Krugman and Venables (1995) and Nocco (2012) among others, and consistent with the empirical observation in Di Giovanni and Levchenko (2010), we assume that there are vertical linkages so firms are using part of the production of the final differentiated good M as an intermediate input. The intermediate composite good used by each firm j is therefore given by

$$M(j) = \left(\int\limits_{l \epsilon \Omega} B_j(l)^{\frac{\sigma-1}{\sigma}} dl \right)^{\frac{\sigma}{\sigma-1}} \quad \sigma > 1$$

where $B_j(l)$ denotes the amount of good l used as intermediate input by the firm producing variety j and σ controls also for the substitution between different intermediate inputs. For simplicity and common to the literature on vertical linkages (i.e., Krugman and Venables (1995), Nocco (2012)) it has been assumed that the elasticity of substitution among final goods in the consumer side is the same as the elasticity of substitution among intermediate inputs in the production function.

To enter into a market a firm needs to invest f_E units of labour to create a new variety. At the moment of entry, the firm is uncertain regarding its productivity although the firm knows that its unit input requirement $a(j)$ follows a Pareto Distribution with the cumulative density function given by $G(a) = \left(\frac{a}{a_M}\right)^{\kappa}$ with $0 \leq a \leq a_M$ and shape parameter $\kappa \geq 1$. After entry, the unit input requirement is revealed to the firm and the firm has the option to leave or stay in the market. Firms that produce must incur a per period fixed cost of operation f_D in terms of the differentiated final good.

Once the firm stays, it has the possibility to export to the foreign market. Exporting, however, entails both fixed and variable trade costs. More precisely, exporters need to incur a per period fixed cost of exporting f_X in terms of the differentiated final good and a variable trade cost of the iceberg type so that $\tau \geq 1$ units of the good have to be shipped from the production country in order to sell one unit in the foreign country. At this stage, we depart from Nocco (2012) assuming that the firm can simultaneously decide to adopt a more efficient Hicks-neutral technology which reduces the marginal cost of production to be a proportion γ of the original value $a(j)$, with $0 < \gamma \leq 1$, so that the unit input requirement of the composite good of firm j that innovates is $\gamma a(j)$. Adopting the most efficient technology bears a cost of f_I units in terms of the final differentiated good.[10]

We assume that the homogenous good is the numeraire. This, together with the assumption of perfect competition, the linear technology (with unit input requirement equal to one) and zero trade cost assumed for this good, imply that the equilibrium wage is equal to one in both countries ($w = 1$). Finally, as it is common in the literature in trade and firm heterogeneity, and to keep the analysis tractable, the paper focuses on the case of symmetric economies (i.e., H and F exhibit identical parameter values, and therefore the solutions for the endogenous variables are symmetric).

2.1 Equilibrium

Solving the consumers' utility maximization problem, the demand function for variety j in the final good sector in each country is given by

$$C(j) = \frac{p(j)^{-\sigma}}{P_M^{1-\sigma}} \mu I$$

where I represents the aggregate income of the country, $p(j)$ the price of variety j and $P_M = \left(\int_0^N p(l)^{1-\sigma} dl \right)^{\frac{1}{1-\sigma}}$ is the price index of the set of all the N differentiated varieties bought in the country, which includes both domestically produced and imported varieties.

To obtain the total demand function that each firm faces we need to specify the different components of it. Specifically, a firm can produce to satisfy the demand of both domestic and foreign consumers and firms. In general, the quantity produced in H by firm j is given by the following expression

$$\begin{aligned} q(j) &= C_D(j) + B_{HDN}(j) + B_{HDI}(j) + B_{HXN}(j) + B_{HXI}(j) + \\ &\quad + \epsilon \left(C_X(j) + B_{FDN}(j) + B_{FDI}(j) + B_{FXN}(j) + B_{FXI}(j) \right) \end{aligned}$$

where: $\epsilon = \tau$ if the firm exports and 0 otherwise; $C_D(j)$ and $C_X(j)$, respectively, denote the demand for variety j for domestic and foreign consumption; the variable $B_{vsm}(j)$ indicates the demand for variety j used as an intermediate input by firms producing in country $v = H, F$ for their domestic market (obtained when $s = D$) and to export to the

[10] Following the notation of Baldwin and Forslid (2004), the parameters f_D, f_X and f_E are, respectively, the discounted value of the fixed cost of producing for the domestic country, for export and entry. These elements consequently are the equivalent to δf_D, δf_X and δf_E in Melitz (2003). Moreover, f_I is the discounted value of the fixed cost of innovation.

other country (when $s = X$). Note that, within each case, the model distinguishes between the demand of the adopters of the most efficient technology (when $m = I$) and the non-adopters (when $m = N$) since the unit input requirements are different across the two cases. An analogous expression holds for the quantity produced by firm j producing in F.

A firm's demand for a variety used as an intermediate input is obtained by applying Shepard's lemma to its total cost function. In general terms, the total cost function of firm ι is given by

$$TC(\iota) = P_M^{\alpha}(f_D + \xi f_X + \zeta f_I + \gamma^{\zeta} a(\iota) q_{\iota})$$

where the variable ξ takes value one if the firm exports and zero otherwise, while ζ takes the value one if the firm innovates and zero otherwise. The demand of firm ι for each variety j is, thus, given by $B_{\iota}(j)$

$$\begin{aligned} B_{\iota}(j) &= \frac{\partial TC(\iota)}{\partial p(j)} = \alpha \frac{p(j)^{-\sigma} P_M^{\alpha}}{P_M^{1-\sigma}} \left(f_D + \xi f_X + \zeta f_I + \gamma^{\zeta} a(\iota) q_{\iota}\right) = \\ &= \frac{p(j)^{-\sigma}}{P_M^{1-\sigma}} \alpha TC(\iota) \end{aligned}$$

Notice that firms behave monopolistically and so the price that firm j charges in the domestic market is

$$p_{Dm}(j) = \left(\frac{\sigma}{\sigma - 1}\right) a(j) \gamma^{\zeta} P_M^{\alpha}$$

In the case in which firm j exports, the price that it sets abroad is $p_{Xm}(j) = \tau p_{Dm}(j)$. Thus, exporting firms set higher prices abroad and firms adopting the most efficient technology charge lower prices in both markets.

In each country the total value of the expenditure in the differentiated manufactured varieties E is given by the sum of the share of consumers' income, μI, and of the share of the total cost of production spent on intermediates, αTC, i.e., $E = \mu I + \alpha TC$. Equilibrium aggregate production for each variety j is, consequently, given by

$$q(j) = (1 + \xi\phi) \left(\frac{p_{Dm}(j)^{-\sigma}}{P_M^{1-\sigma}}\right) E$$

where $\phi \equiv \tau^{1-\sigma}$ denotes the freeness of trade. This parameter takes a value from zero to one and it is increasing as trade is less costly. When the trade costs go to infinity, this parameter will goes to zero, and this will be the autarkic case. When $\tau = 1$, there are no trade costs and international trade is free with $\phi = 1$.

A firm's operating profits with unit input requirement $a(j)$ are then given by

$$\pi(j) = (1 + \xi\phi) \Delta \gamma^{\zeta(1-\sigma)} a(j)^{1-\sigma}$$

where $\Delta \equiv \left(\frac{E}{\sigma (P_M)^{(1-\alpha)(1-\sigma)}}\right) \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}$ captures aggregate variables and parameters affecting firms' demand. In the case when $\alpha > 0$, this term also includes parameters affecting its production cost levels. The term $\gamma^{\zeta(1-\sigma)}$ denotes a measure of the efficiency gains obtained by the firm when it adopts the most efficient technology (when $\zeta = 1$), with larger gains obtained for smaller values of γ.[11]

A firm with unit input requirement $a(j)$ decides to upgrade its state of technology when the benefits of adopting the new technology expressed in terms of larger operating profits overcome the costs of doing so, that is when

$$(1 + \xi\phi)\Delta\left(\gamma^{1-\sigma} - 1\right) a(j)^{1-\sigma} \geqslant f_I P_M^{\alpha}$$

with the sign of equality holding if the firm is indifferent between adopting or not adopting the new technology. Its unit input requirements, a_I, will be denoted as the adoption cutoff.

A firm decides to export if the operating profits obtained from the foreign market overcome the fixed cost of exporting, that is if

$$\phi\Delta\gamma^{\zeta(1-\sigma)} a(j)^{1-\sigma} \geqslant f_X P_M^{\alpha}$$

with the sign of equality holding if the firm is indifferent between exporting or not. The indifferent exporter is called the marginal exporter and its unit input requirement is denoted by a_X.

The firm that is indifferent between staying or leaving the market must satisfy the following condition

$$\Delta\gamma^{\zeta(1-\sigma)} a_D^{1-\sigma} = f_D P_M^{\alpha}$$

where a_D denotes its unit input requirements.

Note that if a firm finds it profitable to adopt the new technology just considering the domestic market ($\Delta(\gamma^{(1-\sigma)} - 1)a^{1-\sigma} \geqslant f_I P_M^{\alpha}$), then this firm must adopt the same technology if it decides to export. This implies that there cannot be simultaneously in an equilibrium domestic firms adopting the most efficient technology and exporters relying on their original technology.

Depending on the parameter configurations, this model exhibits three different types of equilibria.[12] Moreover, different types of equilibria are associated with different firm hierarchies regarding export and innovation activities.

If *adopting the most productive technology* is an expensive activity relative to exporting, that is if the following

[11] As smaller values of γ imply larger values of $\gamma^{(1-\sigma)}$

[12] See Navas and Sala (2015) for a complete characterization of all types of equilibria in a model without vertical linkages. The existence of vertical linkages does not alter the conditions determining the parameter configuration associated with each type of equilibria. This comes from the fact that external linkages affect both export and innovation activities in a similar way as they affect the production for the domestic market given that all fixed costs are using intermediates with the same intensity.

condition holds

$$\frac{f_I}{(\gamma^{1-\sigma}-1)}\left(\frac{\phi}{1+\phi}\right) > f_X > \phi f_D,$$

firms will be sorted according to the following status: adopting exporters (the most productive ones), non adopting exporters with intermediate productivity levels and domestic firms (the least productive ones). In this equilibrium, technology adoption is a relatively expensive activity (compared to exporting) and only a subset of the most productive exporters are willing to upgrade the technology they use. We denote this equilibrium by A.

If, instead, *exporting* is a relatively expensive activity the following condition holds

$$f_X > \frac{f_I}{(\gamma^{1-\sigma}-1)}\phi\gamma^{1-\sigma} > \phi\gamma^{1-\sigma} f_D,$$

and firms are grouped according to the following categories: exporting adopters (the most productive ones), non exporting adopters (intermediate productivity levels) and domestic firms (the least productive ones). We denote this equilibrium by B.

Finally, there is another case for intermediate trade costs in which firms do not find it profitable to upgrade the technology they use if they do not export and viceversa. More precisely, this equilibrium is sustained when the following parameter configuration holds

$$\frac{f_I}{(\gamma^{1-\sigma}-1)}\frac{\phi}{(1+\phi)} \leq f_X \leq \frac{f_I}{(\gamma^{1-\sigma}-1)}\gamma^{1-\sigma}\phi.$$

In this equilibrium firms are sorted according to two different types: adopters that export (the most productive ones) or just domestic firms (the least productive ones). We denote this equilibrium by C.[13]

In the following subsections we characterize the different equilibrium conditions for each type of equilibrium. We denote the respective unit input cutoffs associated with each equilibrium with the superscript $i = \{A, B, C\}$.

2.1.1 Equilibrium A

In equilibrium A every firm that adopts is an exporter but only a subset of the most productive exporters adopt. Consequently the conditions defining the three unit input cutoffs ranked as $a_I^A < a_X^A < a_D^A$, explained above, are given by

$$(1+\phi)\Delta^A(\gamma^{1-\sigma}-1)\left(a_I^A\right)^{1-\sigma} = f_I\left(P_M^A\right)^{\alpha} \tag{1}$$

$$\phi\Delta^A\left(a_X^A\right)^{1-\sigma} = f_X\left(P_M^A\right)^{\alpha} \tag{2}$$

[13] Note that $\frac{f_I}{(\gamma^{1-\sigma}-1)}\frac{\phi}{(1+\phi)} \leq \frac{f_I}{(\gamma^{1-\sigma}-1)}\gamma^{1-\sigma}\phi$, since $\frac{1}{1+\phi} \leq \gamma^{1-\sigma}$.

$$\Delta^A \left(a_D^A\right)^{1-\sigma} = f_D \left(P_M^A\right)^{\alpha} \tag{3}$$

A firm's decision to adopt the new technology, is affected by the presence of vertical linkages through two channels. The first one affects clearly a firm's demand through Δ^A. In a model without vertical linkages, total expenditure includes only domestic and foreign consumer expenditure. In this model, however, total expenditure is determined not only by consumer expenditure but also by each firm's demand for varieties used as intermediates. In principle, firms can rely on more incentives to adopt the most efficient technology since market size is larger. The existence of vertical linkages also has an impact on the firm's marginal costs of production, having an impact on global sales and operating profits through the price index P_M affecting Δ^A when $\alpha > 0$. This second channel, which is absent in a model without vertical linkages, also affects the cost of adoption. As the fixed costs of adoption involve the use of intermediates, any change in the cost of intermediates affects the adoption cost.

Using (1) and (3), the proportion of surviving firms which adopt the most efficient technology is obtained as

$$\frac{N_I^A}{N_D^A} = \left(\frac{a_I^A}{a_D^A}\right)^{\kappa} = \left[\frac{(1+\phi)(\gamma^{1-\sigma}-1)f_D}{f_I}\right]^{\frac{\kappa}{\sigma-1}} \tag{4}$$

where N_I^A and N_D^A are, respectively, the number of innovating and surviving firms.

Using (2) and (3) the proportion of surviving firms exporting is

$$\frac{N_X^A}{N_D^A} = \left(\frac{a_X^A}{a_D^A}\right)^{\kappa} = \left(\frac{\phi f_D}{f_X}\right)^{\frac{\kappa}{\sigma-1}} \tag{5}$$

with N_X^A denoting the number of exporting firms.

2.1.2 Equilibrium B

In equilibrium B, the firm that is indifferent between adopting or relying on the original technology is a domestic firm and all exporters are in fact adopters, with the three unit input cutoffs ranked as $a_X^B < a_I^B < a_D^B$. In this case the following conditions hold

$$\phi\gamma^{1-\sigma}\Delta^B \left(a_X^B\right)^{1-\sigma} = f_X \left(P_M^B\right)^{\alpha} \tag{6}$$

$$(\gamma^{1-\sigma}-1)\Delta^B \left(a_I^B\right)^{1-\sigma} = f_I \left(P_M^B\right)^{\alpha} \tag{7}$$

$$\Delta^B \left(a_D^B\right)^{1-\sigma} = f_D \left(P_M^B\right)^{\alpha} \tag{8}$$

Similar conclusions relative to the impact of vertical linkages on the decision of adopting the most efficient technology can be extracted. Analogous to the previous equilibrium, using (9) and (8) the proportion of surviving firms exporting is given by

$$\frac{N_X^B}{N_D^B} = \left(\frac{a_X^B}{a_D^B}\right)^{\kappa} = \left(\frac{\phi\gamma^{1-\sigma} f_D}{f_X}\right)^{\frac{\kappa}{\sigma-1}} \tag{9}$$

and using (10) and (8) the proportion of firms adopting the most efficient technology is given by,

$$\frac{N_I^B}{N_D^B} = \left(\frac{a_I^B}{a_D^B}\right)^{\kappa} = \left[\frac{\left(\gamma^{1-\sigma} - 1\right) f_D}{f_I}\right]^{\frac{\kappa}{\sigma-1}} \tag{10}$$

2.1.3 Equilibrium C

In equilibrium C, the set of adopters and exporters coincide. The firm which is indifferent between adopting or relying on the original technology knows that if it does not adopt, it won't be able to export and viceversa. Consequently, that firm evaluates the benefits of jointly adopting and exporting instead of relying with the original technology and remaining domestic. The two resulting cutoffs are ranked as follows $a_I^C < a_D^C$, where a_I^C denotes the unit input requirements associated with that firm. The conditions associated with this equilibrium are given by the following expressions

$$\Delta^C[(1+\phi)\gamma^{(1-\sigma)} - 1]\left(a_I^C\right)^{1-\sigma} = (f_I + f_X)\left(P_M^C\right)^{\alpha}. \tag{11}$$

$$\Delta^C a_D^{1-\sigma} = f_D \left(P_M^C\right)^{\alpha} \tag{12}$$

Dividing (11) and (12) and rearranging terms, the proportion of surviving firms which adopt and export is given by:

$$\frac{N_X^C}{N_D^C} = \frac{N_I^C}{N_D^C} = \left(\frac{a_I^C}{a_D^C}\right)^{\kappa} = \left\{\frac{\left[(1+\phi)\gamma^{1-\sigma} - 1\right] f_D}{f_I + f_X}\right\}^{\frac{\kappa}{\sigma-1}} \tag{13}$$

Note that the presence of vertical linkages does not have an impact on the productivity distribution conditional on entry in any of the three equilibria, (i.e., the proportion of surviving firms adopting the most efficient technology or exporting is unchanged by the presence of vertical linkages). This is the result of both channels affecting symmetrically the variable profits of adopting, exporting and staying in the market. This can be observed by looking at conditions (4) and (5) in equilibrium A, (9) and (10) in equilibrium B and (13) in equilibrium C and noting that neither Δ^i nor $\left(P_M^i\right)^{\alpha}$ are affecting these proportions.[14] However, the presence of vertical linkages has interesting implications for

[14] Given that the expression for P_M^i is different across equilibria because it depends on θ^i defined in the following subsection, Δ^i changes across equilibria and it should therefore be changed accordingly.

the survival productivity cut-off having a clear impact on the technology adoption cut-off and, ultimately, the average industry productivity and the welfare of the economy.

2.2 Solution

An equilibrium in this economy is characterized by a vector of unit input cutoffs (a_I^i, a_X^i, a_D^i), and a vector of aggregate variables (P_M^i, E^i, N_D^i, N_X^i, N_I^i) that satisfy the specific equations associated with each equilibrium described above, the market clearing conditions for each good and the Free Entry condition.

The conditions derived in each of the equilibria above reveal that a_I^i, a_X^i can be expressed as a function of a_D^i. The value of a_D^i can be obtained as in the Melitz (2003) model using the Zero Profit condition (ZP) and the Free Entry condition (FE). The ZP condition is given by[15]

$$\Delta^i \left(a_D^i\right)^{1-\sigma} = f_D \left(P_M^i\right)^{\alpha} \tag{14}$$

and the FE is given by

$$\bar{\pi}^i = \left(P_M^i\right)^{\alpha} f_D \theta^i + \frac{f_E}{G(a_D^i)}, \quad i = A, B, C.$$

where $\bar{\pi}^i$ represents average operating profits, $\theta^i \equiv \left[1 + \frac{G(a_I^i)}{G(a_D^i)} \frac{f_I}{f_D} + \frac{G(a_X^i)}{G(a_D^i)} \frac{f_X}{f_D}\right]$ and, in the specific case of equilibrium C, the cutoffs are such that $a_X^C = a_I^C$. The free entry condition states that average expected operating profit of active producers must be equal to their expected fixed cost $\left(P_M^i\right)^{\alpha} f_D \theta^i$ - which is given by the sum of $\left(P_M^i\right)^{\alpha} f_D$, plus $\left(P_M^i\right)^{\alpha} f_I$ times the probability of being an adopter (conditional on it being a producer), plus $\left(P_M^i\right)^{\alpha} f_X$ times the probability of being an exporter (again, conditional on it being a producer) - plus the expected cost of developing a successful entrant, that is $f_E/G(a_D^i)$.

Given that $\bar{\pi}^i = \bar{r}^i/\sigma$, where $\bar{r}^i$ represents the average revenues, substituting $\bar{r}^i = E^i/N_D$, we find that the free entry condition can be rewritten as

$$\frac{E^i}{\sigma N_D} = \left(P_M^i\right)^{\alpha} f_D \theta^i + \frac{f_E}{G(a_D^i)} \tag{15}$$

To solve the model, an expression for the aggregate price index P_M^i and one for total expenditure E^i must be obtained. Using their definitions, we obtain respectively the price index and the expenditure:

$$P_M^i = \left(\frac{\sigma}{\sigma - 1} a_D^i\right)^{\frac{1}{(1-\alpha)}} \left(\frac{\beta}{\beta - 1}\right)^{\frac{1}{(1-\alpha)(1-\sigma)}} \left(N_D^i\right)^{\frac{1}{(1-\alpha)(1-\sigma)}} \left(\theta^i\right)^{\frac{1}{(1-\alpha)(1-\sigma)}} \tag{16}$$

$$E^i = \mu L + \alpha \left(P_M^i\right)^{\alpha} N_D^i f_D \left[1 + \frac{(\sigma - 1)\kappa}{\kappa - (\sigma - 1)}\right] \theta^i \tag{17}$$

[15] Note that this corresponds to the following respective conditions for each equilibria (3), (8), (12).

where $\beta \equiv \frac{\kappa}{\sigma-1} > 1$.[16] Conditions (14)-(17) characterize a system of equations in 4 endogenous variables (a_D^i, P_M^i, N_D^i and E^i). Solving the system we find that:

$$a_D^i = \left[\frac{\kappa\mu L}{\delta_0 f_D}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\right]^{\frac{\alpha}{\chi}}\left[\frac{a_M^\kappa f_E}{f_D}\frac{(\beta-1)}{\theta^i}\right]^{\frac{\sigma-1-\alpha\sigma}{\chi}} \qquad (18)$$

$$P_M^i = \left[\frac{\kappa\mu L}{\delta_0 f_D}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\right]^{-\frac{\kappa}{\chi}}\left[\frac{a_M^\kappa f_E}{f_D}\frac{(\beta-1)}{\theta^i}\right]^{\frac{\sigma-1}{\chi}} \qquad (19)$$

$$N_D^i = \frac{\left(\frac{\beta-1}{\beta}\right)\left(\frac{\sigma}{\sigma-1}\right)^{(\sigma-1)}\left[\frac{\kappa\mu L}{\delta_0 f_D}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\right]^{(\sigma-1)\frac{\alpha+\kappa(1-\alpha)}{\chi}}}{\left[\frac{f_E}{f_D}(\beta-1)\,a_M^\kappa\right]^{\alpha\frac{\sigma-1}{\chi}}\left[\theta^i\right]^{\kappa\frac{\sigma-\alpha\sigma-1}{\chi}}}$$

where $\chi \equiv (\sigma-1)(\alpha+\kappa) - \alpha\kappa\sigma$ and $\delta_0 \equiv \alpha(\sigma-1) + \kappa\sigma(1-\alpha) > 0$. Note that χ is positive when $\alpha \in [0, \alpha_1)$ and negative when $\alpha \in (\alpha_1, 1)$, with $\alpha_1 \equiv \kappa/(\beta\sigma-1) < 1$, while θ^i is given by:

$$\theta^i = \left\{\begin{array}{ll} 1 + \phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta} + \left(\gamma^{1-\sigma}-1\right)^\beta(1+\phi)^\beta\left(\frac{f_I}{f_D}\right)^{1-\beta} & \text{if } i = A \\ 1 + \left(\phi\gamma^{1-\sigma}\right)^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta} + \left(\gamma^{1-\sigma}-1\right)^\beta\left(\frac{f_I}{f_D}\right)^{1-\beta} & \text{if } i = B \\ 1 + \left[(1+\phi)\gamma^{1-\sigma}-1\right]^\beta\left(\frac{f_X+f_I}{f_D}\right)^{1-\beta} & \text{if } i = C \end{array}\right\} \qquad (20)$$

Note that with any change in trade costs (variable or fixed), the efficiency gains from technology adoption, γ, and its fixed cost have an impact on the survival unit input cut-off through the variable θ^i. Observing the equilibrium expression above one can conclude that θ^i can be interpreted as an indicator of trade openness and innovation opportunities generated by the levels of international integration and the conditions characterizing the innovative environment in the industry. Higher θ^i (i.e., lower variable trade costs, larger efficiency gains from technology adoption and/or lower fixed costs of exporting or technology adoption) has contrasting effects: from one side it expands the business opportunities of the most productive firms *(near the highest marginal cutoffs)*, but from the other side it intensifies competition for every firm toughening the conditions for surviving in the market. Eventually, this implies a more selective industry.

The analysis of (20) reveals that θ^i is not affected by the existence of vertical linkages. Vertical linkages, however, play a role on the effects on the survival unit input cut-off and on welfare through the price index P_M^i.

3 The impact of trade liberalization on unit input cutoffs.

Table 1 summarizes and represents with arrows the effects of trade liberalization, that consists of a reduction in variable trade costs (and thus increases ϕ), on the different unit input cutoffs and on welfare (this latter effect will be discussed

[16] This condition is required to have the price index P_M^i converging to a positive value.

	Welfare	**Equilibrium A**		**Equilibrium B**	**Equilibrium C**	
$0 < \alpha < \alpha_0$	$W \uparrow$	$a_D^A \downarrow$	$a_I^A \uparrow$	a_D^B and $a_I^B \downarrow$	$a_D^C \downarrow$	$a_I^C \uparrow$
$\alpha_0 < \alpha < \alpha_1$	$W \uparrow$	$a_D^A \uparrow$	$a_I^A \uparrow$	a_D^B and $a_I^B \uparrow$	$a_D^C \uparrow$	$a_I^C \uparrow$
$\alpha_1 < \alpha < \alpha_2^i$	$W \downarrow$	$a_D^A \downarrow$	$a_I^A \downarrow$	a_D^B and $a_I^B \downarrow$	$a_D^C \downarrow$	$a_I^C \downarrow$
$\alpha_2^i < \alpha < 1$	$W \downarrow$	$a_D^A \downarrow$	Case 1 $a_I^A \downarrow$ Case 2 $a_I^A \uparrow$	a_D^B and $a_I^B \downarrow$	$a_D^C \downarrow$	Case 1 $a_I^C \downarrow$ Case 2 $a_I^C \uparrow$

Table 1: The effects of an increase in the freeness of trade on the productivity and innovation cutoffs

in next Section).

In the table: $\alpha_0 \equiv \frac{\sigma-1}{\sigma}$, $\alpha_1 \equiv \frac{\kappa}{\beta\sigma-1}$, α_2^i is equal to $\alpha_2^A \equiv \frac{\kappa(\theta^A-(1+\phi))}{\phi(1-\beta\sigma)+(\phi+\beta\sigma)(\theta^A-1)}$ for Equilibrium A and it is equal to $\alpha_2^C \equiv \frac{\kappa}{\beta\sigma-\theta^C}$ for Equilibrium C. [17] Note that for Equilibria A and C different effects are produced by the increase in ϕ only on a_I^A and a_I^C when vertical linkages are very large, that is when $\alpha_2^i < \alpha < 1$, and that for Equilibrium B there is no threshold α_2^B as a unique case occurs for $\alpha > \alpha_1$.

The baseline case, that is the one without vertical linkages ($\alpha = 0$), has the same qualitative effects as those described by the first row in the table above with low intensity of vertical linkages. Specifically, in the case without vertical linkages, trade liberalization increases (decreases) the proportion of surviving firms that undertake technology adoption in equilibrium A and C (equilibrium B).

When the economy experiences a process of trade liberalization, the benefits of technology adoption are affected by two opposing forces: Firstly, firms enjoy an expansion in their business opportunities due to better access to the foreign market. This is reflected by the multiplicative term, $(1+\phi)$, in conditions (1) and (11) which is positively associated with ϕ, the freeness of trade. This is the traditional market size effect found in innovation models.[18] As the market size increases, the firm is able to apply the reduction in marginal costs associated with the new technology across more production units increasing the benefits of adoption.

However, together with the increase in market size, an increase in ϕ also reduces the potential sales of the firm in each market. This is because as trade costs are reduced, this improves the access of foreign firms into the domestic market. As each firm needs to compete with more firms within each market for consumers, their sales, ceteris paribus, are reduced in each market. This effect is captured in conditions (1) and (11) by the element $\left(P_M^i\right)^{1-\sigma}$, which depends positively on θ^i. An increase in ϕ increases θ^i reducing the firms' market share within each market. The net effect will be the result of these two opposing forces and for the case of the equilibrium A and C the net effect on the benefits of technology adoption is positive, therefore increasing the mass of firms that technology upgrade. Equation (3) helps us to conclude that the latter effect reduces the proportion of firms that survive in the market.

[17] We show how the values of α_2^i can be determined in the Appendix where we derive also the signs of the derivatives $\frac{\partial a_D^i}{\partial \phi}$ and $\frac{\partial a_I^i}{\partial \phi}$ used to establish the effects of increases in ϕ on the cutoffs

[18] See Grossman and Helpman (1991), Peretto (2016), Impullitti and Licandro (2016), Acemoglu (2009) and more recently Aghion et al (2017) among others.

In equilibrium B, instead, the marginal adopter is not an exporter. This implies that this firm suffers from the fall in the domestic sales due to the competition from foreign firms in the domestic market but it does not enjoy the expansion of rents from foreign markets. Consequently, in this case, trade liberalization will reduce the firms' adoption profits and therefore this will reduce the incentives to technology upgrade. Likewise, the reduction in operating profits due to the fall in domestic sales reduces the proportion of firms surviving in the market.

The presence of vertical linkages modifies some of the channels discussed above while it brings some new forces at play. Let us consider first the case in which vertical linkages are small ($0 < \alpha < \alpha_0$) and without loss of generality the equilibrium B.[19] In this case, substituting Δ^B into (7) gives

$$\frac{E^B \left(P_M^B\right)^{\alpha(1-\sigma)}}{\sigma \left(P_M^B\right)^{(1-\sigma)}} \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left(a_D^B\right)^{1-\sigma} = f_D \left(P_M^B\right)^{\alpha} \tag{21}$$

and the condition that determines the adoption cutoff in this equilibrium is given by

$$(\gamma^{1-\sigma}-1)\frac{E^B \left(P_M^B\right)^{\alpha(1-\sigma)}}{\sigma \left(P_M^B\right)^{(1-\sigma)}} \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left(a_I^B\right)^{1-\sigma} = f_I \left(P_M^B\right)^{\alpha} \tag{22}$$

Vertical linkages affect the production costs and the fixed costs of adoption, export and survival. Trade liberalization reduces the cost of imports, reducing the aggregate price index P_M^B. Since final goods are used as inputs in the production process, the reduction in the cost of imports reduces production costs, the variable ones (captured by the term $\left(P_M^B\right)^{\alpha(1-\sigma)}$ in the left hand side of (21) and (22)) and the fixed operational ones (captured by the element $\left(P_M^B\right)^{\alpha}$ in the right hand side of (21) and (22)). These two effects have a positive effect on the survival unit input cut-off a_D^B and the adoption cut-off a_I^B.[20] In addition vertical linkages also has a positive effect on each variety's demand which also increases survival (This effect is included in the element E^B in the model) and it will intensify competition. All of these effects are shaped by the parameter α which measures the strength of vertical linkages. The larger is this parameter, the more the production of the firm will be dependent on the goods produced by other firms (local and foreign). In that case, a reduction of trade costs will have a larger impact on the channels mentioned above. In the case discussed in the current paragraph, α is low, and therefore the negative effect on domestic sales captured by the element $\left(P_M^B\right)^{(1-\sigma)}$ in the denominator dominates. This implies that the qualitative effects of trade liberalization will be unchanged compared to a scenario in which vertical linkages are not present.

However, these results are challenged when the intensity of vertical linkages is medium ($\alpha_0 < \alpha < \alpha_1$). When the intensity of vertical linkages is middle-low, trade liberalization still reduces the aggregate price index. Nevertheless, in

[19] Note that condition (1) differs from (10) only in the multiplicative term $(1+\phi)$ which is not affected by vertical linkages, while conditions (3) and (8) are identical. Therefore, for our discussion, we can analyse the contribution of vertical linkages to the effects of trade liberalisation on innovation and survival by focusing on equilibrium B and extend these arguments to equilibrium A. A similar reasoning applies for equilibrium C.

[20] Note that P_M^B will be affected by trade costs, through the direct effect on export prices and through the indirect effect on the costs of intermediates.

this case, vertical linkages are strong enough to have a positive impact on survival and technology adoption: the initial reduction in firms' domestic sales is dominated by the effect that the reduction in the aggregate price index has on the firm's production costs and the demand effect. Because production becomes cheaper, firms' operating profits increase and this will allow the least productive firms to survive: this clearly increases the survival unit input cut-off. The reduction in the costs of production and adoption will increase the benefits of technology adoption and it will reduce the fixed costs of adoption. This will exert a double impact on innovation: on the one hand a larger proportion of surviving firms become innovators (only for equilibria A and C) and, on the other hand, trade liberalization increases the cutoff of firms surviving in the market (common to all the three type of equilibria). Overall the proportion of entrants adopting the most efficient technology increases after trade liberalization in all equilibria. This result is novel and thus not present in a model without vertical linkages.

When the degree of vertical linkages is relatively high $\alpha_1 < \alpha < \alpha_2$, the opposite happens. Not only does trade liberalization have a negative impact in the unit input cut-off (which makes firms' survival more difficult), but also the effect is strong enough to reduce the cutoff of firms that adopt the most efficient technology. Condition (19) reveals that trade liberalization increases the aggregate price index. Condition (16) could help us to grasp an intuition behind this result. This condition reveals that, holding constant a_D^i and N_D^i, trade liberalization increases θ^i and reduces the level of the aggregate price index. This reduces production costs and it increases the demand for intermediate inputs, pushing upwards their prices. This will have an impact on the survival cutoff a_D^i and the surviving number of firms N_D^i. The overall general equilibrium effect taking into account the effects in all variables is an increase in the equilibrium price index. The increase in the equilibrium price index increases the production costs and the fixed costs of innovation, export and survival, inducing a lower proportion of firms to survive and adopt the most efficient technology. While the increase in the aggregate price index as a result of general equilibrium effects is already present in Nocco (2012), the decrease in the proportion of firms technology upgrading in all equilibria is a novel result that it is not present in a model without vertical linkages.[21]

In the extreme cases in which α is very high $\alpha_2 < \alpha < 1$, the impact of trade liberalization on survival and technology adoption depends clearly on the parameters of the model. When the economy is in equilibrium A or in equilibrium C we can distinguish two cases: one in which trade liberalization deters technology adoption (Case 1) and one in which trade liberalization promotes technology adoption (Case 2). Case 1 arises when the initial level of international integration is already relatively high (either because the fixed cost of exporting is relatively small or because trade costs are relatively low, or both of them are small) or when the initial level of international integration is small but associated with a small fixed cost of adoption and/or a large reduction in unit requirements as a result of technology adoption. In both situations the pressures on demand for goods and intermediates are already *large* and a reduction in trade costs

[21] Navas and Sala (2015) analyses the impact of trade on technology adoption in a similar framework without vertical linkages. In that model, in equilibrium A and C trade liberalisation will always increase the proportion of adopting firms in the economy.

puts a lot of pressure on demand that increases the price index of goods and deters technology adoption. Case 2, on the contrary, arises when the initial level of international integration is relatively low (either because the fixed cost of exporting is relatively large or because trade costs are relatively high, or both of them are large) and this is associated with a high fixed cost of adoption of a new technology and/or with small gains in productivity levels associated with new technology. In this case the pressures on the demand for goods and intermediates are smaller as *initially* fewer firms innovate due to the large cost of technology adoption and/or the small potential productivity gain. Thus, a larger proportion of very productive firms would profit from trade liberalization, adopting the new technology even though the price index of intermediates is rising. Moreover, in the Appendix we discuss special parameter configurations under which these two cases arise independently of the value of ϕ.[22]

4 Effects on Welfare

The previous section shows that vertical linkages shape the effect that trade liberalization has on technology adoption. In this section we analyze the importance of vertical linkages on the effects of trade liberalization on welfare focusing on the technology adoption channel. To do so, we first analyze the impact of trade liberalization on welfare when firms are allowed to technology upgrade and vertical linkages are (not) present. In a second exercise, we compare the effects of trade liberalization on welfare in a model with vertical linkages in which firms are (not) allowed to technology upgrade. This strategy allows us to better capture the specific contribution of each channel to the effect of trade liberalization on welfare. To illustrate the quantitative importance of these two dimensions for the impact of trade liberalization on welfare we simulate the model for different degrees of vertical linkages. Our main conclusion is that both channels exacerbate the impact of trade liberalization on welfare and for the case with vertical linkages the ultimate effect of trade liberalization on welfare depends crucially on their strength (α). The simulation results suggest that these effects are also quantitatively important.

4.1 Theoretical results

Substituting the optimal values for C_T and C_M, the indirect utility function can be expressed as

$$U = \frac{L}{\left(P_M^i\right)^{\mu}}$$

Thus, as it is standard in this literature, to evaluate the impact of trade liberalization on welfare it is useful to see how the aggregate price index changes with respect to changes in trade policy.[23]

[22] See the subsection "Discussion on cases 1 and 2 found when $\alpha_2^i < \alpha < 1$" of the Appendix.

[23] As the utility function is Cobb Douglas, the standard expression for their indirect utility function is $U = \frac{I}{(P_M^i)^{\mu}(P_T^i)^{1-\mu}}$, where I represents the economy's aggregate income and $P_T = 1$ since good T is the numeraire. Note that in this model the free entry condition guarantees that in equilibrium the firms' aggregate profits are zero and so the total income is given by wL where $w = 1$.

The following proposition states that trade liberalization has a positive effect on welfare if and only if the degree of vertical linkages are not too strong.

Proposition 1 *Trade liberalization has a positive (non positive) impact on welfare when* $0 \leq \alpha < \alpha_1$ $(\alpha \geqslant \alpha_1)$.
Proof. *See Appendix.* ■

Nocco (2012) establishes that trade liberalization has a negative impact on welfare when the degree of vertical linkages is relatively strong ($\alpha > \alpha_1$) as trade liberalization increases the price index of the composite good. Table 1 corroborates that this continues to be the case in this scenario when firms are allowed to technology upgrade. The main reason behind that result lies in the positive effect that trade liberalization has on the demand for the composite good, because firms' willingness to become exporters increases. This increases the costs of surviving and entry, reducing the mass of varieties produced in equilibrium. In the particular context of the present model, the effect could be stronger for some cases since, apart from the effect on the mass of varieties, the increase in the cost of intermediates can have, in certain cases, a negative impact on technology upgrading.

While the effects of trade liberalization on welfare are qualitatively the same as in Nocco (2012), this cannot be said from a quantitative point of view. The following two propositions state that the inclusion of both technology upgrading and vertical linkages magnify the effects of trade liberalization on welfare.

Proposition 2 *The presence of vertical linkages strenghtens the impact that trade liberalization has on welfare.*

Proof. *See Appendix.* ■

The existence of vertical linkages generates a multiplier effect on welfare. This multiplier effect comes through the direct impact of trade liberalization on the price index, which declines with moderate vertical linkages ($0 < \alpha < \alpha_1$), and the specific effects of trade liberalization on selection and technology upgrading that vary according to the strength of vertical linkages and type of equilibria. In a standard model of trade with technology upgrading (i.e., Navas and Sala (2015)), trade liberalization has an impact on the price of the final composite good, through, first, the increase in the mass of varieties, N, (i.e., the variety channel), second, through the reduction in the cost per imported variety (i.e., the cost-reduction channel), and third through the effects that trade has on selection and technology adoption. The presence of vertical linkages will magnify the last two channels. As the final goods are used as intermediate inputs, the reduction in trade costs will further reduce the final good prices through the reduction in the costs of intermediates. In addition, vertical linkages through general equilibrium effects have an impact on selection and technology adoption, although this impact will vary with the type of equilibria, and the strength of vertical linkages. Focusing on equilibrium A, when vertical linkages are weak, ($0 < \alpha < \alpha_0$), trade liberalization reinforces selection and encourages more firms to technology upgrade. Both channels contribute to a decrease in the aggregate price index. When vertical linkages are intermediate-low ($\alpha_0 < \alpha < \alpha_1$) the interaction between selection and technology adoption becomes more complex as trade liberalization softens selection but boosts technology adoption. When $\alpha_0 < \alpha < \alpha^* = \frac{(\sigma-1)^2+(\sigma-1)\kappa}{(\sigma-1)^2+\sigma\kappa}$, the increase in technology adoption more than compensates the negative effect that trade liberalization has on selection (selection is softened) and overall we will observe a decline in the aggregate price index. When $\alpha^* < \alpha < \alpha_1$, the effect on selection dominates although the overall effect taking into account the variety, the cost-reduction and the technology adoption channel will be positive.[24][25] In a similar way, the increase in the costs of intermediates that comes because of the increase in the price index generated by trade liberalization when $\alpha > \alpha_1$ will lead to magnification effects on welfare.

In addition to the multiplier effect that we find under the presence of vertical linkages, the welfare effects of trade liberalization are exacerbated in this framework due to the possibility of firms to technology upgrade. Notice that in a model without vertical linkages (as in Nocco (2012)), the equivalent to θ^i is $\tilde{\theta} = 1+\phi^{\beta}\left(\frac{f_X}{f_D}\right)^{1-\beta}$. Since in the Appendix we show that the effects of trade liberalization are determined by the elasticity of θ^i to τ the following can be concluded:

Proposition 3 *The welfare effects of trade liberalization are magnified when technology upgrading is possible.*

Proof. *See Appendix* ■

Expression (20) shows that θ^i is larger in the technology adoption case. In addition, the effects of trade liberalization on welfare depends on how trade costs affect θ^i, which is also larger in the three cases.[26]

The main reason behind this result lies in the fact that trade liberalization promotes technology upgrading in equilibria A and C and this subsequently reduces the price index, having a larger impact on welfare. In equilibrium B, trade liberalization, while deterring technology upgrading in certain cases (i.e., $0 < \alpha < \alpha_0$), facilitates the replacement of domestic varieties by more productive foreign ones. This increases welfare.

This section has shown that the inclusion of both vertical linkages and the possibility of firms to upgrade technology magnifies the effects of trade liberalization on welfare. This section has also shown that under certain conditions trade liberalization may have a negative impact on welfare. The next section reveals that the inclusion of these two new channels has also important quantitative implications.

4.2 Simulations

In the previous paragraphs the magnification effect of vertical linkages and technology upgrading in the impact of trade liberalization on welfare has been established theoretically. An important question to address is to what extent these effects are quantitatively relevant. In this subsection this question is tackled by considering the effects of trade liberalization on welfare and other variables by examining the impact of a reduction in iceberg-type trade costs from

[24] See the Appendix for a computation of the average productivity of the industry ($\tilde{a}^{1-\sigma}$) and how this varies depending on the degree of vertical linkages. Changes in the average productivity are at the heart of the intuition provided above for the case of $\alpha_0 < \alpha < \alpha_1$.

[25] When $\alpha^* < \alpha < \alpha_1$, in the Appendix it is shown that trade liberalisation reduces the average productivity $\tilde{a}^{1-\sigma}$. The overall fall in the aggregate price index comes in this context because less varieties die with trade liberalisation (the fall in N becomes smaller).

[26] For further details, see the Appendix.

75% to 15%.[27] To undertake these exercises, the parameters regarding the elasticity of substitution and the Pareto distribution follow the empirical results using plant level data reported in Bernard, Jensen Eaton and Kortum (2003), and, therefore, $\sigma = 3.8$, $\kappa = 3.4$ and $a_M = 5$. The rest of the production parameters follow Bernard, Redding and Schott (2004) except for the fixed cost of exporting which is set equal to the fixed cost of entry (and larger than the fixed operational cost) to allow for selection into exporting when trade costs are zero. For the case of the adoption fixed cost and the productivity increase, the parameters chosen respectively are $f_I = 20$ and $\gamma = 0.8$. Given these parameter values, the economy is characterized by an equilibrium of type A.[28] Within each equilibria, several robustness checks have been undertaken to understand the relevance of these parameter values. No significant differences have been obtained in these exercises.[29] Note that, with these parameters, the middle-low level of vertical linkages threshold is $\alpha_0 = 0.73$ and the middle-high level threshold is given by $\alpha_1 = 0.94$.

Figure 3a shows the effects of trade liberalization on the probability of surviving in the industry (i.e., as a percentage change). As pointed out in the section above, when the vertical linkages are low (that is $\alpha < 0.73$), trade liberalization reinforces selection, worsening the probability of survival by 27%. It is interesting to point out that, provided that vertical linkages are not middle high (that is $\alpha < 0.94$), as vertical linkages become more important (that is, α increases) selection is softened and trade liberalization has a smaller percentage impact on selection. This is because the cost-reduction effect yielded by vertical linkages becomes more important, as discussed in section 3. When vertical linkages are middle-low with $0.73 < \alpha < 0.94$, trade liberalization softens selection. More precisely, trade liberalization increases the survival probability by 2.8% when $\alpha = 0.75$ and 64% when $\alpha = 0.85$). However, and although we consider that there could be limited cases in which this result could be applied in the real world, when the vertical linkages are middle-high ($\alpha = 0.95$), the impact of trade liberalization is to reinforce selection again and in this case the probability of surviving becomes 100% lower when trade costs are cut in the magnitude discussed above.[30]

Moreover, Figure 3b shows that trade liberalization boosts technology adoption in this equilibrium as long as vertical linkages are not middle-high and that the impact of trade liberalization on technology adoption becomes larger when the strength of vertical linkages increases. Specifically, when vertical linkages are not present ($\alpha = 0$) the increase in the probability to innovate as a consequence of trade liberalization is only 9%. However when $\alpha = 0.4$, the increase almost doubles and becomes equal to 15%. In the case of middle-low vertical linkages the increase becomes considerably

[27] Anderson and Van Wincoop (2004) suggest that on average, industrialized countries face total trade costs of 21% (tax equivalent). Bernard Jensen and Schott (2006) suggest that, for the case of the US manufacturing sector, the average trade costs manufacturing sector is 8%. The figures provided consider the impact of trade liberalization on welfare from a very high level of trade costs to a rough average of these two measures.

[28] The seminal paper by Bustos (2011) found empirical relevance for this type of equilibria in her study of technology upgrading following MERCOSUR bilateral trade liberalization between Brazil and Argentina in the early 90s. However, we have let the fixed cost of adoption to vary allowing for the other two types of equilibria to appear. In equilibrium B the contribution of these two channels towards the impact of trade liberalization on welfare is slightly reduced from a quantitative point of view. These results are available upon request.

[29] Robustness checks are available upon request.

[30] Jones (2011) offer data on a country's average intermediate input share computed using the OECD input-output database. The average intermediate input share is 0.52 with China experiencing the highest value (0.68). Di Giovanni and Levchenko (2010) uses the UNIDO database to provide data on the average (across countries) sectoral intermediate input share. The data reveal substantial differences across industries. Considering as intermediate input share the value of output needed from all sectors (including its own sector) to produce one dollar of final output, the average is 0.58 with a maximum of 0.948 (Transport equipment).

high (i.e., 52% when $\alpha = 0.75$ and 65% when $\alpha = 0.85$). Yet, when vertical linkages are middle-high ($\alpha = 0.95$), trade liberalization will reduce rather than increase the probability of adoption and it will do it considerably (a reduction of 100%).

Insert Figures 3a and 3b

In addition, Figures 4a and 4b show the percentage changes in average productivity and welfare of an undercut in trade costs. Note that as the strength of vertical linkages increases, the impact on the average productivity (Figure 4a) is smaller. This is the result of two contrasting effects at work. Indeed, as vertical linkages increase, the percentage change in the probability of adoption increases (since adoption becomes cheaper) and this positively affects the average productivity. However the impact on selection is the opposite (i.e., the percentage change in the probability of surviving decreases) as survival becomes easier and this negatively affects the average productivity. Overall, what this exercise suggests is that the effect on selection dominates and that as the vertical linkages are gaining importance, the overall impact on average productivity coming from trade liberalization is smaller. Note, however, that, as long as the vertical linkages are low, the effects are very similar across different degrees of vertical linkages and large (being the increase in average productivity as large as 77%). When the vertical linkages are middle-high ($\alpha = 0.75$), however, the increase in productivity is much smaller (33%). Although the increase in technology adoption is substantially larger, not only trade liberalization will not contribute to increase the average productivity through selection but it will contribute to reduce it because selection is softened rather than toughened. The larger the vertical linkages, the softer selection will be. Note that when $\alpha = 0.85$, as a matter of fact, trade contributes negatively to the average productivity because the impact of selection is far larger than the impact on technology adoption.

Note that unlike the average productivity, the gains from trade liberalization in terms of welfare in figure 4b are indeed increasing with the degree of vertical linkages provided that the degree of vertical linkages are not middle high. Moreover, the existence of vertical linkages magnify the effects of trade liberalization. When vertical linkages are not present, trade liberalization generates an increase in welfare of approximately 6%. With a moderate increase in vertical linkages, the gains from trade increase considerably. If $\alpha = 0.4$, a value which is slightly below cross-country average value for vertical linkages suggested by Jones (2011), the gains from trade will almost double (10.10%). If the economy enters in the middle-low equilibrium ($\alpha > 0.73$), the increase in welfare will be even larger (33% if $\alpha = 0.75$ and 77% if $\alpha = 0.85$). The figure also shows that when the vertical linkages are middle-high, as shown in our paper, welfare indeed decreases.

In principle, the fact that the gains from trade liberalization on welfare increase with vertical linkages may seem paradoxical with the fact that the increase in the average productivity following trade liberalization decreases as vertical linkages become more important. The key to understanding this result lies in the response of the economy to trade liberalization in the number of varieties. Figure 4c helps us to reconcile this evidence by showing that the decline in domestic varieties is smaller as vertical linkages become more important. As it can be observed, the decline in the number of varieties under the absence of vertical linkages ($\alpha = 0$) is substantial. However, this fall becomes less important in number as the vertical linkages increase. As a matter of fact trade liberalization will increase the number of varieties rather than decrease for the case when vertical linkages are middle-low with $\alpha = 0.85$. It is precisely the smaller decline in domestic varieties that will bid the prices of the production factors up (through the demand effect) with the consequence of a decrease in both the survival and technology adoption cutoff for the case when vertical linkages are middle-high.

Insert Figures 4a, 4b and 4c

To examine the role played by technology adoption in the gains from trade, we first compute the effects of trade liberalization on the average productivity and welfare in a model with and without innovation. Then, we compute the difference between both magnitudes. Figures 5a and 5b show the contribution of technology adoption to the effects of trade liberalization on the average productivity and on welfare. Specifically, figure 5a, displays the contribution of technology adoption to the effects of trade liberalization on the average productivity. The effect of trade liberalization on welfare in a model with technology adoption is 12% larger than in a model without technology adoption when there are no vertical linkages. Given that, the increase in the average productivity from trade liberalization in a model without technology adoption is 65%, we can conclude that including technology adoption rises the gains in average productivity from trade liberalization by 18%. However as vertical linkages increase the contribution of technology adoption to the effects of trade liberalization on average productivity shrinks in percentage points. This, as a matter of fact, is the consequence of the negative impact of vertical linkages on the evolution of the average productivity due to the fact that as vertical linkages are more important, selection is softened. This latter effect is much more important than the effect on innovation. As a matter of fact when the vertical linkages are low, increasing the strenght of vertical linkages does not change much the gains in the average productivity in percentage changes. It only falls substantially to 16% ($\alpha = 0.75$) and 13% ($\alpha = 0.85$) when vertical linkages are middle-low.

Figure 5b shows the contribution in percentage points of technology adoption to the effects of trade liberalization on welfare. Note that in a situation in which there are no vertical linkages the contribution is not very substantial. In a model with technology adoption, the gains from trade liberalization are 0.71 percentage points larger. Although this looks like a small effect , taking into account that our model will predict a gain from trade liberalization in terms of welfare of 4.96% when no technology upgrading is present, the latter constitutes an increase in the gains from trade liberalization of 14%. Yet, as vertical linkages become stronger this effect magnifies in terms of percentage points and slightly in terms of percentage increase. If vertical linkages go from $\alpha = 0$ to $\alpha = 0.4$ then the increase in the effect of trade liberalization on welfare becomes 1.3% which is almost double with respect to the scenario with no vertical linkages, although the percentage increase is exactly the same. If the vertical linkages are middle-low the difference becomes more important with an increase in the gains from trade liberalization from 4.3% with $\alpha = 0.75$ to 12% with

$\alpha = 0.85$. In percentage changes, the increase in the gains from trade on welfare are respectively 16% and 18.50%.

Insert Figures 5a and 5b

5 Conclusion

In this paper we have analyzed the impact of trade liberalization on technology adoption, average productivity and welfare in a model with heterogeneous firms that can decide to upgrade the state of technology. In addition these firms are interconnected by vertical linkages. The joint effect of these two channels and the fact that they can be intertwined is a relatively unexplored dimension in the literature. Our analysis is motivated by the prominent role of intermediate inputs in world trade and GDP and the importance of industry-specific vertical linkages as documented in Di Giovanni and Levchenko (2010).

We have shown that the inclusion of vertical linkages changes the effects of trade liberalization on technology adoption, average productivity and welfare although its final impact depends on the strength of vertical linkages, generating a very rich set of results. When the strength of vertical linkages is low, vertical linkages augment the effects of trade liberalization on technology upgrading under certain conditions. When the strength of vertical linkages are middle-low, instead, trade liberalization increases technology upgrading independently of the parameter configuration. In this case trade liberalization reduces the survival and the costs of technology upgrading allowing more firms to stay and to adopt the new technology. However, when the strength of vertical linkages is middle-high trade liberalization reduces both technology upgrading and the industry cutoff independently of the parameter configuration. For very high levels of vertical linkages the effect on technology adoption depends on the level of economic integration and other parameters. This rich set of results is absent in a model with technology upgrading when vertical linkages are not present

These results have a clear impact on average productivity and welfare. Specifically, trade liberalization increases average productivity, except when $\alpha^* < \alpha < \alpha_1$. In this case, trade liberalization promotes technology adoption but it softens selection. The latter effect dominates contributing to a decrease in average productivity. We also show that trade liberalization has a positive impact on welfare only when vertical linkages are moderate (i.e., $0 < \alpha < \alpha_1$). When vertical linkages are very strong ($\alpha > \alpha_1$) the reduction in the mass of varieties increases the aggregate price index and decreases utility even if there is an increase in average productivity due to a stronger selection effect.

Finally, our framework also has implications for the magnitude of the effects of trade on both welfare and average productivity. More precisely, we have shown theoretically that vertical linkages magnify these effects. Our illustrative quantitative analysis suggests that these magnification effects are also quantitatively important.

Thus, we conclude that our paper helps to shed some light on the analysis of the relationships that exist among trade liberalization, technology adoption, productivity and welfare, showing how its multifaceted nature can become even more complex when firms producing in open economies are interconnected by vertical linkages.

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6 Appendix

6.1 Results in Table I.

6.1.1 Derivations of the results in Table 1

This part of the Appendix contains the derivation of the signs of the derivatives of the cutoffs with respect to ϕ reported in Table I.

In general, from (18) and (20) it can be derived that

$$\frac{\partial a_D^i}{\partial \phi} = -\left(\frac{\sigma - 1 - \alpha\sigma}{\gamma}\right)\left[\frac{\kappa\mu L}{\delta_0 f_D}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\right]^{\frac{\alpha}{\chi}}\left[\frac{a_M^\kappa f_E}{f_D}(\beta-1)\right]^{\frac{\sigma-1-\alpha\sigma}{\chi}}\left(\theta^i\right)^{-\frac{\sigma-1-\alpha\sigma}{\chi}-1}\frac{\partial\theta^i}{\partial\phi}$$

Given that $\frac{\partial\theta^i}{\partial\phi} > 0$, the sign of $\frac{\partial a_D^i}{\partial\phi}$ depends on the sign of $-\frac{\sigma-1-\alpha\sigma}{\chi} = \frac{\alpha\sigma-(\sigma-1)}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}$ as in Nocco (2012). Hence, $\frac{\partial a_D^i}{\partial\phi} < 0$ for $\alpha \in (0, \alpha_0)$ and $\alpha \in (\alpha_1, 1)$, and viceversa $\frac{\partial a_D^i}{\partial\phi} > 0$ for $\alpha \in (\alpha_0, \alpha_1)$ with $\alpha_0 \equiv \frac{\sigma-1}{\sigma}$, $\alpha_1 \equiv \frac{\kappa}{\beta\sigma-1}$ and $0 < \alpha_0 < \alpha_1 < 1$.

Turning to the sign of $\frac{\partial a_I^i}{\partial\phi}$, it has to be analyzed in detail for each case.

Equilibrium A In equilibrium A, substituting a_D^A into (4) yields

$$a_I^A = z\left[\left(\theta^A\right)^{-\frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}}\left(\frac{1}{1+\phi}\right)^{\frac{1}{(1-\sigma)}}\right]$$

where $z \equiv \left[\frac{\mu\kappa}{\delta_0 f_D}L\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\right]^{\frac{\alpha}{\chi}}\left\{\left[\frac{f_I}{f_D}\frac{1}{(\gamma^{1-\sigma}-1)}\right]^{\frac{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}{(1-\sigma)(\sigma-1-\alpha\sigma)}}\frac{f_E}{f_D}(\beta-1)\,a_M^\kappa\right\}^{\frac{\sigma-1-\alpha\sigma}{\chi}} > 0$ does not depend on ϕ.

Then,

$$\frac{\partial a_I^A}{\partial\phi} = -z\left(\theta^A\right)^{-\frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}}\left(\frac{1}{1+\phi}\right)^{\frac{1}{(1-\sigma)}}\frac{\frac{\partial\theta^A}{\partial\phi}}{\theta^A}(h-l)$$

and given that $z\left(\theta^A\right)^{-\frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}}\left(\frac{1}{1+\phi}\right)^{\frac{1}{(1-\sigma)}}\frac{\frac{\partial\theta^A}{\partial\phi}}{\theta^A} > 0$, the sign of $\frac{\partial a_I^A}{\partial\phi}$ depends on the sign of $h-l$, where $h \equiv \frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}$ and $l \equiv \frac{1}{(\sigma-1)}\frac{1}{1+\phi}/(\frac{\frac{\partial\theta^A}{\partial\phi}}{\theta^A})$. It can be readily verified that l does not depend on α and can be represented by a horizontal line in the cartesian plane where α is represented on the axis of abscissas (See Figures 1-2 that follow). Moreover, when the independent variable is α, h is a hyperbola, whose expression can be rewritten as follows $h = \frac{\sigma\alpha-(\sigma-1)}{[\kappa\sigma-(\sigma-1)]\alpha-(\sigma-1)\kappa}$ with center in $O\left(\frac{\kappa}{\beta\sigma-1}, \frac{1}{\kappa-\frac{(\sigma-1)}{\sigma}}\right)$, and its two asymptotes having, respectively, expressions $\alpha_1 \equiv \frac{\kappa}{\beta\sigma-1}$ in the case of the vertical asymptote and $h_1 = \frac{1}{\kappa-(\sigma-1)/\sigma}$ in that of the horizontal asymptote. Moreover, it can be verified that: 1) $h = \frac{1}{\kappa}$ when $\alpha = 0$; 2) $h = \frac{1}{\kappa-(\sigma-1)} > 0$ (because $\kappa > \sigma - 1$) when $\alpha = 1$; 3) and, finally, $h = 0$ when $\alpha = \frac{\sigma-1}{\sigma} \equiv \alpha_0$.

Then, substituting $\frac{\frac{\partial \theta^A}{\partial \phi}}{\theta^A}$ into l yields

$$l = \frac{1}{\kappa}\frac{\phi}{1+\phi}\frac{1+\phi^{\beta}\left(\frac{f_X}{f_D}\right)^{1-\beta}+\left(\gamma^{1-\sigma}-1\right)^{\beta}(1+\phi)^{\beta}\left(\frac{f_I}{f_D}\right)^{1-\beta}}{\phi^{\beta}\left(\frac{f_X}{f_D}\right)^{1-\beta}+\left(\gamma^{1-\sigma}-1\right)^{\beta}(1+\phi)^{\beta}\frac{\phi}{1+\phi}\left(\frac{f_I}{f_D}\right)^{1-\beta}} > \frac{1}{\kappa}$$

with $l < \frac{1}{\kappa-(\sigma-1)}$ for sufficiently high levels of international integration, that is with $(f_X/\phi)^{\beta-1} < \frac{(\beta+\phi)}{(\beta-1)} f_D^{\beta-1}$. Otherwise, for low levels of international integration, that is with $(f_X/\phi)^{\beta-1} > \frac{(\beta+\phi)}{(\beta-1)} f_D^{\beta-1}$, we find that: $l < \frac{1}{\kappa-(\sigma-1)}$ if $\left(\frac{1}{f_I}\right)^{\beta-1} > \frac{\phi(\beta-1)-\Omega(\beta+\phi)}{(\gamma^{1-\sigma}-1)^{\beta}(1+\phi)^{\beta}\phi f_D^{\beta-1}}$ with $\Omega \equiv \phi^{\beta}\left(\frac{f_X}{f_D}\right)^{1-\beta}$; and $l > \frac{1}{\kappa-(\sigma-1)}$ if $\left(\frac{1}{f_I}\right)^{\beta-1} < \frac{\phi(\beta-1)-\Omega(\beta+\phi)}{(\gamma^{1-\sigma}-1)^{\beta}(1+\phi)^{\beta}\phi f_D^{\beta-1}}$.

Therefore, there are two potential cases:

Case 1 $l < \frac{1}{\kappa-(\sigma-1)}$ when $(f_X/\phi)^{\beta-1} < \frac{(\beta+\phi)}{(\beta-1)} f_D^{\beta-1}$ or when $(f_X/\phi)^{\beta-1} > \frac{(\beta+\phi)}{(\beta-1)} f_D^{\beta-1}$ and $\left(\frac{1}{f_I}\right)^{\beta-1} > \frac{\phi(\beta-1)-\Omega(\beta+\phi)}{(\gamma^{1-\sigma}-1)^{\beta}(1+\phi)^{\beta}\phi f_D^{\beta-1}}$. In this case represented in Figure 1, $\frac{\partial a_I^A}{\partial \phi} > 0$ when $0 \leq \alpha < \alpha_1$ as $h < l$; and $\frac{\partial a_I^A}{\partial \phi} < 0$ when $\alpha_1 < \alpha < 1$ as $h > l$.

Insert Figure 1 about here

Case 2 $l > \frac{1}{\kappa-(\sigma-1)}$ when $(f_X/\phi)^{\beta-1} > \frac{(\beta+\phi)}{(\beta-1)} f_D^{\beta-1}$ and $\left(\frac{1}{f_I}\right)^{\beta-1} < \frac{\phi(\beta-1)-\Omega(\beta+\phi)}{(\gamma^{1-\sigma}-1)^{\beta}(1+\phi)^{\beta}\phi f_D^{\beta-1}}$. In this case, $\frac{\partial a_I^A}{\partial \phi} > 0$ when $0 \leq \alpha < \alpha_1$ and when $\alpha_2^A < \alpha < 1$ as $h < l$; and $\frac{\partial a_I^A}{\partial \phi} < 0$ when $\alpha_1 < \alpha < \alpha_2^A$ as $h > l$.

Insert Figure 2 about here

Equilibrium C In equilibrium C, substituting a_D^C into (13) yields

$$a_I^C = y\left(\theta^C\right)^{-\frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}}\left[\frac{1}{(1+\phi)\gamma^{1-\sigma}-1}\right]^{\frac{1}{(1-\sigma)}}$$

where $y \equiv \left[\frac{\kappa\mu L}{f_D\delta_0}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\right]^{\frac{\alpha}{\chi}}\left(\frac{f_X+f_I}{f_D}\right)^{\frac{1}{1-\sigma}}\left[\frac{a_M^{\kappa} f_E}{f_D}(\beta-1)\right]^{\frac{\sigma-1-\alpha\sigma}{\chi}} > 0$ does not depend on ϕ.

Then,

$$\frac{\partial a_I^C}{\partial \phi} = -y\left(\theta^C\right)^{-\frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}}\left[\frac{1}{(1+\phi)\gamma^{1-\sigma}-1}\right]^{\frac{1}{(1-\sigma)}}\frac{\frac{\partial \theta^C}{\partial \phi}}{\theta^C}(h-l_C)$$

and given that $y\left(\theta^C\right)^{-\frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}}\left[\frac{1}{(1+\phi)\gamma^{1-\sigma}-1}\right]^{\frac{1}{(1-\sigma)}}\frac{\frac{\partial \theta^C}{\partial \phi}}{\theta^C} > 0$, the sign of $\frac{\partial a_I^C}{\partial \phi}$ depends on the sign of $h - l_C$, where $l_C \equiv \frac{1}{(\sigma-1)}\frac{\gamma^{1-\sigma}}{(1+\phi)\gamma^{1-\sigma}-1}/(\frac{\frac{\partial \theta^C}{\partial \phi}}{\theta^C})$ does not depend on α and can be represented by an horizontal line in the cartesian plane where α is represented on the axis of abscissas.

Then, substituting $\frac{\frac{\partial \theta^C}{\partial \phi}}{\theta^C}$ into l_C yields

$$l_C = \frac{1}{\kappa}\frac{1+\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}\left(\frac{f_X+f_I}{f_D}\right)^{1-\beta}}{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}\left(\frac{f_X+f_I}{f_D}\right)^{1-\beta}} > \frac{1}{\kappa}$$

with $l_C < \frac{1}{\kappa-(\sigma-1)}$ when $(f_X+f_I)^{\beta-1} < \frac{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}}{(\beta-1)} f_D^{\beta-1}$ (Case 1), and $l_C > \frac{1}{\kappa-(\sigma-1)}$ when $(f_X+f_I)^{\beta-1} > \frac{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}}{(\beta-1)} f_D^{\beta-1}$ (Case 2). In Case 1, that is for relatively low (f_X+f_I) and relatively high levels of economic integration ϕ, $\frac{\partial a_I^C}{\partial \phi} > 0$ when $0 < \alpha < \alpha_1$ as $h < l_C$, while $\frac{\partial a_I^C}{\partial \phi} < 0$ when $\alpha_1 < \alpha < 1$ because $h > l_C$. In Case 2, that is for relatively high (f_X+f_I) and relatively low levels of economic integration ϕ, $\frac{\partial a_I^C}{\partial \phi} > 0$ when $0 < \alpha < \alpha_1$ and $\alpha_2^C < \alpha < 1$ as $h < l_C$, while $\frac{\partial a_I^C}{\partial \phi} < 0$ when $\alpha_1 < 1 < \alpha_2^C$, because $h > l_C$. Notice that the two cases can be represented by similar figures as those respectively used in Figure 1 and Figure 2, even if the expression of l_C should be used instead of that of l and the value of α_2^C should be used instead of that of α_2^A.

Equilibrium B For the case of equilibrium B, it is clear from (10) that $sign(\frac{\partial a_I^B}{\partial \phi}) = sign(\frac{\partial a_D^B}{\partial \phi})$.

6.1.2 Discussion on cases 1 and 2 found when $\alpha_2^i < \alpha < 1$

In the main text we have discussed that when $\alpha_2^i < \alpha < 1$ two different cases arise in equilibrium A and C and we have provided an intuition on when these two cases arise. In addition, the next proposition state parameter configurations in which the economy is in equilibrium A and it is in either case 1 or case 2 independently of the degree of variable trade costs:

Proposition 4 *Consider Equilibrium A with* $\alpha_2^A < \alpha < 1$. *if* $\left(\frac{f_X}{f_D}\right)^{\beta-1} > \frac{\beta+1}{\beta-1}$ *and* $\left(\gamma^{1-\sigma}-1\right) < \beta - 1$ *or* $\frac{f_I}{f_D} > \frac{\left(\gamma^{1-\sigma}-1\right)^{\beta} 2^{\beta}}{(\beta-1)-\left(\frac{f_X}{f_D}\right)^{1-\beta}(\beta+1)}$ *the economy is in case 2 independently of the value of* ϕ.

Proof. In equilibrium A the economy is in case number 2 if $\left(\frac{f_X}{\phi f_D}\right)^{\beta-1} > \frac{(\beta+\phi)}{(\beta-1)}$ and $\left(\frac{f_I}{f_D(\gamma^{1-\sigma}-1)(1+\phi)}\right)^{\beta-1} > \frac{\phi\left(\gamma^{1-\sigma}-1\right)(1+\phi)}{(\phi(\beta-1)-\Omega(\beta+\phi))}$ with $\Omega = \phi^{\beta}\left(\frac{f_X}{f_D}\right)^{1-\beta}$. Otherwise the economy is in case number 1 (which represents the continuity from the previous equilibria). Rearranging terms in the first of the previous conditions we have that $\left(\frac{f_X}{f_D}\right)^{\beta-1} > \phi^{\beta-1}\frac{(\beta+\phi)}{(\beta-1)}$. The right hand side is increasing in ϕ. Since ϕ is upper bounded by 1 we have that the first condition is always satisfied when $\left(\frac{f_X}{f_D}\right)^{\beta-1} > \frac{(\beta+1)}{(\beta-1)}$. Rearranging the second equation and replacing the value of Ω, we have that $\left(\frac{f_I}{f_D}\right)^{\beta-1} > \frac{\left(\gamma^{1-\sigma}-1\right)^{\beta}(1+\phi)^{\beta}}{(\beta-1)-\left(\frac{f_X}{f_D}\right)^{1-\beta}(\beta+\phi)\phi^{\beta-1}}$. The right hand side of this expression is also increasing in ϕ. Since ϕ is upper bounded by 1 we have that the second condition is always satisfied when $\left(\frac{f_I}{f_D}\right)^{\beta-1} > \frac{\left(\gamma^{1-\sigma}-1\right)^{\beta} 2^{\beta}}{(\beta-1)-\left(\frac{f_X}{f_D}\right)^{1-\beta}(\beta+1)}$. Notice that from (4) $\left(\frac{f_I}{f_D(\gamma^{1-\sigma}-1)(1+\phi)}\right)^{\beta-1} > 1$ *must be satisfied* for equilibrium A to hold. This implies that if $\frac{\phi\left(\gamma^{1-\sigma}-1\right)(1+\phi)}{(\phi(\beta-1)-\Omega(\beta+\phi))} < 1$ for any value of ϕ this condition will be always sustained independently of the value of ϕ. Making use of the expression for Ω and rearranging terms we have that for the previous condition to hold $\left(\gamma^{1-\sigma}-1\right) < \frac{\left((\beta-1)-\left(\frac{f_X}{f_D}\right)^{1-\beta}\phi^{\beta-1}(\beta+\phi)\right)}{(1+\phi)}$. Notice that this expression is decreasing in ϕ. Evaluating the rhs under $\phi = 0$ we have that: $\left(\gamma^{1-\sigma}-1\right) < (\beta-1)$. If this condition holds, then $\left(\frac{f_I}{f_D}\right)^{\beta-1} > \frac{\left(\gamma^{1-\sigma}-1\right)^{\beta}(1+\phi)^{\beta}}{(\beta-1)-\left(\frac{f_X}{f_D}\right)^{1-\beta}(\beta+\phi)\phi^{\beta-1}}$ for any value of ϕ. ■

The following proposition discuss parameter configurations in which the economy is in equilibrium C and it is in either case 1 or case 2.

Proposition 5 *Consider Equilibrium C with $\alpha_2^C < \alpha < 1$, then:*[31]

$$\text{if}\left\{\begin{array}{ll} \left(\frac{f_I+f_X}{f_D}\right)^{\beta-1} > \frac{\left(2\gamma^{1-\sigma}-1\right)^{\beta}}{\beta-1} & \textit{the economy is in case 2} \\ \frac{\left(\gamma^{1-\sigma}-1\right)^{\beta}}{\beta-1} < \left(\frac{f_I+f_X}{f_D}\right)^{\beta-1} < \frac{\left(2\gamma^{1-\sigma}-1\right)^{\beta}}{\beta-1} & \textit{the equilibrium of the economy} \\ & \textit{depends on the value of } \phi \\ \left(\frac{f_I+f_X}{f_D}\right)^{\beta-1} < \frac{\left(\gamma^{1-\sigma}-1\right)^{\beta}}{\beta-1} & \textit{the economy is in case 1} \end{array}\right\}$$

Proof. The equation governing the conditions under which the economy will be in either case 1 or 2 is the following: If $\left\{\frac{(f_I+f_X)}{[(1+\phi)\gamma^{1-\sigma}-1]f_D}\right\}^{\beta-1} > \frac{(1+\phi)\gamma^{1-\sigma}-1}{\beta-1}$, the economy is in case 2, otherwise the economy is in case 1 (provided that $\left\{\frac{(f_I+f_X)}{[(1+\phi)\gamma^{1-\sigma}-1]f_D}\right\}^{\beta-1} > 1$ which is required from (13) for being in equilibrium C). Rearranging terms we find that for the former condition to hold the following must be satisfied: $\left[\frac{(f_I+f_X)}{f_D}\right]^{\beta-1} > \frac{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}}{\beta-1}$.Notice that the right hand side of this condition is increasing in ϕ. Because ϕ is bounded above and below, evaluating the rhs of that condition when $\phi = 1$ we have that if $\left[\frac{(f_I+f_X)}{f_D}\right]^{\beta-1} > \frac{\left(2\gamma^{1-\sigma}-1\right)^{\beta}}{\beta-1}$, then $\left[\frac{(f_I+f_X)}{f_D}\right]^{\beta-1} > \frac{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}}{\beta-1}$ for any ϕ (we are in case 2 independently of ϕ). Consider the reverse constraint $\left[\frac{(f_I+f_X)}{f_D}\right]^{\beta-1} < \frac{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}}{\beta-1}$ (this is the condition for being in case 1). Evaluating the right hand side when $\phi = 0$ we have that if $\left[\frac{(f_I+f_X)}{f_D}\right]^{\beta-1} < \frac{(\gamma^{1-\sigma}-1)^{\beta}}{\beta-1}$ then $\left(\frac{(f_I+f_X)}{f_D}\right)^{\beta-1} < \frac{\left((1+\phi)\gamma^{1-\sigma}-1\right)^{\beta}}{\beta-1}$ for any ϕ. The economy is in case 1 independently of the value of ϕ. ■

The previous propositions conclude that for cases in which the fixed costs of exporting or technology adoption are relatively high, trade liberalization will increase the proportion of firms undertaking technology upgrading when the degree of vertical linkages are very strong in equilibria A and C and in the specific case 2. Consequently, the results suggest that while in case 1, trade liberalization induces tougher selection, and reduces the proportion of entrants undertaking technology adoption, in case 2 the latter is not so strong and the overall positive effect dominates. This is consistent with the fact that in case 2 the requirements for innovating and exporting are so large that very few firms decide to undertake these activities. This moderates the rise in the demand of intermediate inputs and consequently the rise in the relative cost of intermediate inputs. The latter has a moderate impact on the survival productivity threshold and the proportion of technology upgrading firms.

6.2 Proof of propositions.

Proof of Proposition 1 Deriving expression (16) with respect to τ we notice that:

$$sign\frac{\partial P_M^i}{\partial \tau} = sign\left(\left(\frac{1-\sigma}{\chi}\right)\left(\frac{\partial \theta^i}{\partial \tau}\right)\right) \tag{23}$$

where we know that $\frac{\partial \theta^i}{\partial \tau} < 0$ since $\frac{\partial \theta^i}{\partial \tau} = \underbrace{\frac{\partial \theta^i}{\partial \phi}}_{+}\underbrace{\frac{\partial \phi}{\partial \tau}}_{-} < 0$. The sign of the derivative of the price index is clearly depending on the sign of $\frac{1-\sigma}{\chi}$, and given that expression $(1-\sigma)$ is always negative, the sign of $\frac{\partial P_M^i}{\partial \tau}$ ultimately depends on the

[31] All of the conditions considered are consistent with the economy being in equilibrium C.

sign of χ that depends on the value of α. More precisely, if $\alpha < \alpha_1$ then χ is positive, while χ is negative if $\alpha > \alpha_1$. Consequently, we obtain that, no matter the equilibrium in which we are, $\frac{\partial P_M^i}{\partial \tau} < 0$ if $\alpha < \alpha_1$ and $\frac{\partial P_M^i}{\partial \tau} > 0$ if $\alpha > \alpha_1$.

Proof of Proposition 2 Consider first the case with no technology adoption and compare the two models with and without vertical linkages. This will be equivalent to (23) with $\alpha = 0$. The effect of trade on welfare in the model without vertical linkages is determined by the effect on the aggregate price index which is given by: $\frac{\partial P_M^*}{\partial \tau} = -\frac{1}{\kappa}\left(\frac{\partial \tilde{\theta}}{\partial \tau}\right)$. $\tilde{\theta}$ is not affected by vertical linkages and, consequently, it will take the same value in the model with vertical linkages. The effect of trade liberalization on welfare in this case is larger in the model with vertical linkages if the following inequality holds $\frac{\sigma-1}{\chi} > \frac{1}{\kappa} \Rightarrow \kappa(\sigma-1) > \chi \Rightarrow 0 > (\sigma-1)\alpha - \alpha\kappa\sigma \Rightarrow \kappa\sigma > \sigma - 1 \Rightarrow \beta > \frac{1}{\sigma}$, which is the case since $\beta > 1$ and $\sigma > 1$.

The same line of reasoning applies in a case in which we consider that firms have the possibility to technology upgrade in both models but vertical linkages are only present in one of them. In that case θ^i instead of $\tilde{\theta}$ will be common across both models. But the difference in the effect of trade liberalization on welfare clearly depends on the same inequality as in the previous case which clearly holds.

Proof of Proposition 3 To show this, first notice that in a model with technology adoption the welfare effect of trade liberalization is given by: $\frac{\partial P_M^i}{\partial \tau} = \frac{1-\sigma}{\chi}\frac{\partial \theta^i}{\partial \phi}\frac{\partial \phi}{\partial \tau}$. In a model without technology upgrading we have that the effect of trade liberalization on welfare is given by: $\frac{\partial P_M^i}{\partial \tau} = \frac{1-\sigma}{\chi}\frac{\partial \tilde{\theta}}{\partial \phi}\frac{\partial \phi}{\partial \tau}$ with $\tilde{\theta} = 1 + \phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta}$. Since we have that $\frac{\partial \theta^i}{\partial \phi} > 0$, $\frac{\partial \tilde{\theta}}{\partial \phi} > 0$, and the other two factors $\frac{1-\sigma}{\chi}$ and $\frac{\partial \phi}{\partial \tau}$ are identical in both models, the effect will be larger in a model with technology adoption when $\frac{\partial(\theta^i - \tilde{\theta})}{\partial \phi} > 0$. Then, let us analyze the three specific equilibria.

Equilibrium A. In equilibrium A, $\theta^i = 1 + \phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta} + \left(\gamma^{1-\sigma} - 1\right)^\beta (1+\phi)^\beta\left(\frac{f_I}{f_D}\right)^{1-\beta}$. Since $\tilde{\theta} = 1 + \phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta}$, then we have that $\theta^i - \tilde{\theta} = \left(\gamma^{1-\sigma} - 1\right)^\beta (1+\phi)^\beta\left(\frac{f_I}{f_D}\right)^{1-\beta} > 0$ increases when ϕ increases. Consequently the impact on welfare is larger in the model with technology upgrading.

Equilibrium B. In equilibrium B, $\theta^i = 1 + \left[\phi\gamma^{1-\sigma}\right]^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta} + \left[\gamma^{1-\sigma} - 1\right]^\beta\left(\frac{f_I}{f_D}\right)^{1-\beta}$. In this case, we have that: $\theta^i - \tilde{\theta} = \left[\gamma^{(1-\sigma)\beta} - 1\right]\phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta} + \left(\gamma^{1-\sigma} - 1\right)^\beta\left(\frac{f_I}{f_D}\right)^{1-\beta} > 0$ since $(1-\sigma)\beta = -\kappa$ is negative and $\gamma \in [0,1]$. Therefore $\theta^i - \tilde{\theta}$ increases when ϕ increases

Equilibrium C. In equilibrium C, $\theta^i = 1 + \left((1+\phi)\gamma^{1-\sigma} - 1\right)^\beta\left(\frac{f_X+f_I}{f_D}\right)^{1-\beta}$. Differentiating with respect to ϕ we have that $\frac{\partial \theta^i}{\partial \phi} = \beta\left[(1+\phi)\gamma^{1-\sigma} - 1\right]^{\beta-1}\gamma^{1-\sigma}\left(\frac{f_X+f_I}{f_D}\right)^{1-\beta} > 0$, and $\frac{\partial \tilde{\theta}}{\partial \phi} = \beta\phi^{\beta-1}\left(\frac{f_X}{f_D}\right)^{1-\beta} > 0$. The third term of the first derivative is larger than one and the fourth term is larger than the third term of the second derivative. To ensure that $\frac{\partial \theta^i}{\partial \phi}$ is larger than $\frac{\partial \tilde{\theta}}{\partial \phi}$, so that $\theta^i - \tilde{\theta}$ increases when ϕ increases, we need to compare $\left[(1+\phi)\gamma^{1-\sigma} - 1\right]^{\beta-1}$ and $\phi^{\beta-1}$. Given that $\beta > 1$, both these two elements have a positive exponent. Then we have that: $(1+\phi)\gamma^{1-\sigma} - 1 > \phi$ if $(1+\phi)\left(\gamma^{1-\sigma} - 1\right) > 0$ which is the case, also ensuring that $\theta^i - \tilde{\theta} > 0$.

6.3 Average productivity

The aggregate price index can be expressed as

$$P_M^i = \left(N_D^i\right)^{\frac{1}{1-\sigma}} p(\tilde{a}^i) = \left(N_D^i\right)^{\frac{1}{1-\sigma}} \left(\frac{\sigma}{\sigma-1}\right) \left(P_M^i\right)^{\alpha} \tilde{a}^i$$

that can be rewritten as

$$P_M^i = \left(N_D^i\right)^{\frac{1}{(1-\sigma)(1-\alpha)}} \left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{1-\alpha}} \left(\tilde{a}^i\right)^{\frac{1}{1-\alpha}}$$

The previous expression together with equation (18) yields average productivity as follows

$$\left(\tilde{a}^i\right)^{1-\sigma} = \left(\frac{\beta}{\beta-1}\right) \theta^i \left(a_D^i\right)^{1-\sigma}$$

The impact of trade liberalization can be assessed analyzing how a change in variable trade barriers affect average productivity. Notice that a reduction in trade barriers will have a positive effect on θ^i . However it has an ambiguous effect on a_D^i depending on the value of α. More precisely we have that

$$\frac{\partial \left(\tilde{a}^i\right)^{1-\sigma}}{\partial \tau} = \frac{\partial \left(\tilde{a}^i\right)^{1-\sigma}}{\partial \theta^i} \frac{\partial \theta^i}{\partial \tau}$$

where $\frac{\partial \theta^i}{\partial \tau} < 0$. The first term is given by:

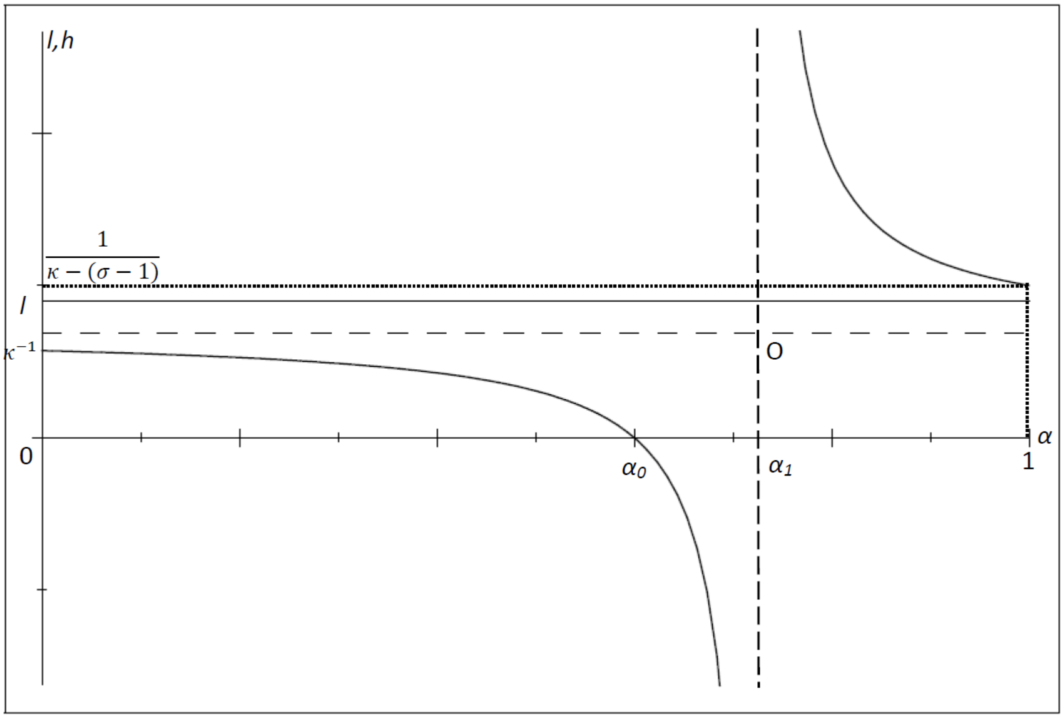
$$\begin{aligned} \frac{\partial \left(\tilde{a}^i\right)^{1-\sigma}}{\partial \theta^i} &= \left(\frac{\beta}{\beta-1}\right) \left(a_D^i\right)^{1-\sigma} \left[1 + \frac{\partial \left(a_D^i\right)^{1-\sigma}}{\partial \theta^i} \frac{\theta^i}{\left(a_D^i\right)^{1-\sigma}}\right] = \\ &= \left(\frac{\beta}{\beta-1}\right) \left(a_D^i\right)^{1-\sigma} \left[1 - (1-\sigma) \frac{(\sigma - 1 - \alpha\sigma)}{\chi}\right] \end{aligned}$$

Notice that the sign of this derivative depends on the last factor in the squared brackets. More precisely the sign of $\frac{\partial \left(\tilde{a}^i\right)^{1-\sigma}}{\partial \theta^i}$ is positive when

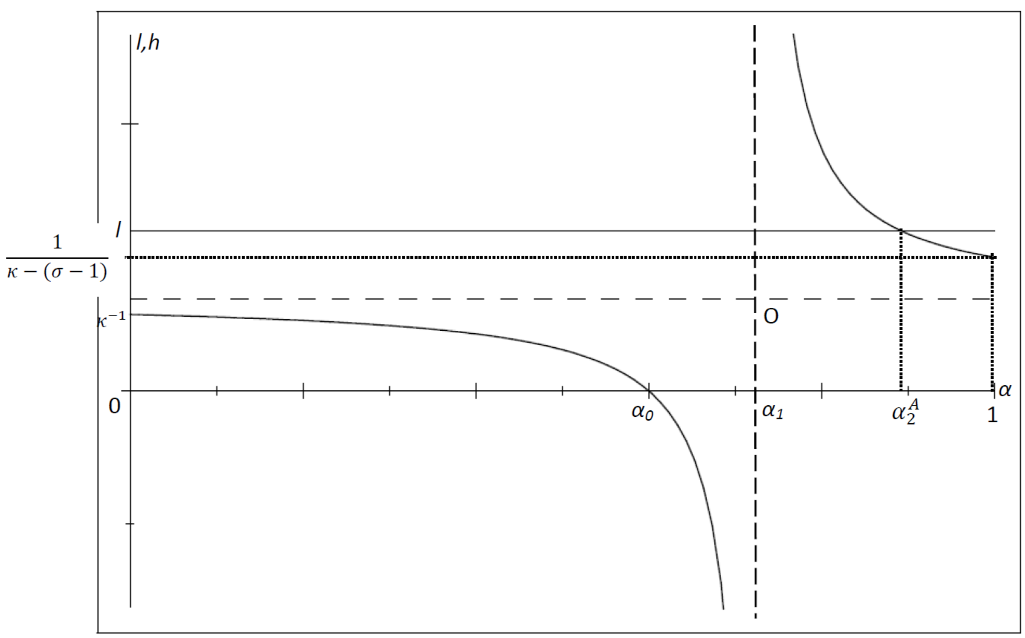
$$\frac{(\sigma-1)(\sigma-1-\alpha\sigma)}{(\sigma-1)(\alpha+\kappa) - \alpha\kappa\sigma} > -1 \tag{24}$$

and negative otherwise. Notice that the denominator $\chi \equiv (\sigma-1)(\alpha+\kappa) - \alpha\kappa\sigma$ is positive when $\alpha \in [0, \alpha_1)$ and negative when $\alpha \in (\alpha_1, 1)$. Hence, it can be shown that if $\alpha \in [0, \alpha_1)$, $\frac{\partial \left(\tilde{a}^i\right)^{1-\sigma}}{\partial \theta^i} > 0$ if $\alpha \in [0, \alpha^*)$ with $\alpha^* \equiv \frac{(\sigma-1)^2 + (\sigma-1)\kappa}{(\sigma-1)^2 + \sigma\kappa}$, and $\frac{\partial \left(\tilde{a}^i\right)^{1-\sigma}}{\partial \theta^i} < 0$ if $\alpha \in (\alpha^*, \alpha_1)$. If, instead, $\alpha \in (\alpha_1, 1)$, χ is negative and given that both the numerator and the denominator of (24) are negative, the inequality in (24) is always true and hence the sign of $\frac{\partial \left(\tilde{a}^i\right)^{1-\sigma}}{\partial \theta^i}$ is always positive.

6.4 Figures



Parameter range for Case 1 in Equilibrium A.



Parameter range for case 2 in equilibrium A.

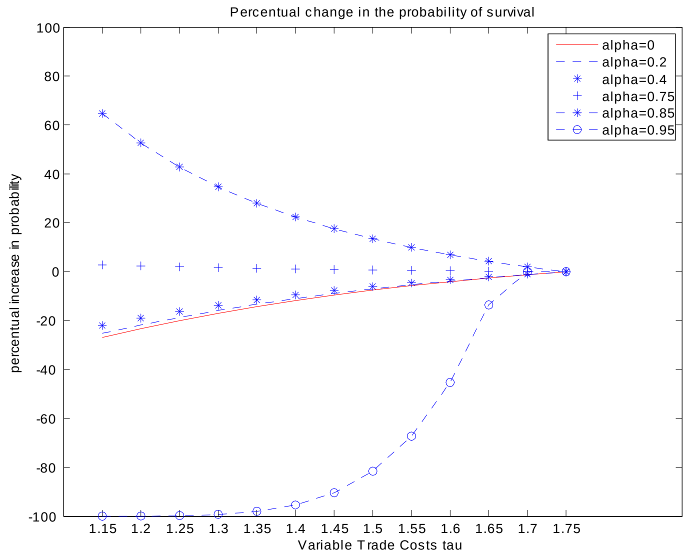


Fig 3a: Effects of trade liberalisation on survival probability

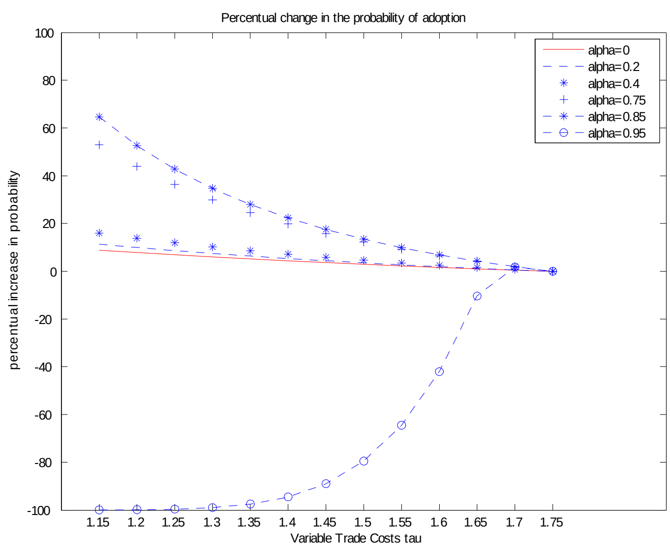


Fig 3b: Effects of trade liberalisation on the probability to technology upgrade

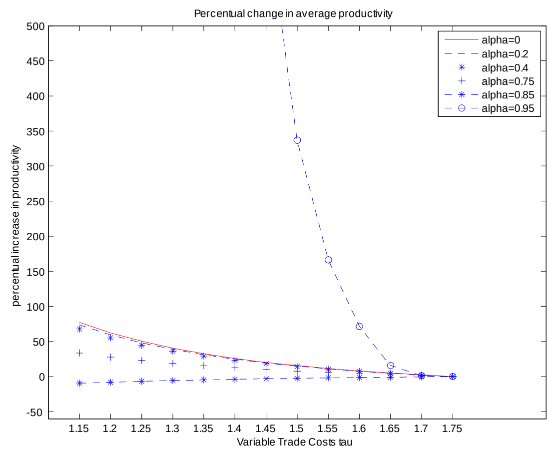


Fig. 4a: The effects of trade liberalisation on average productivity

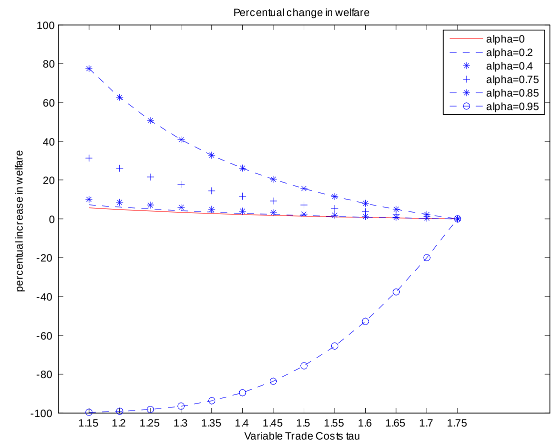


Fig 4b: The effects of trade liberalisation on welfare.

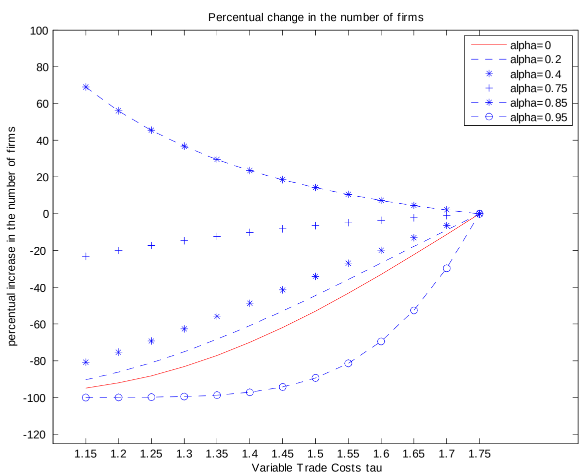


Fig 4c: The effects of trade liberalisation on the number of varieties

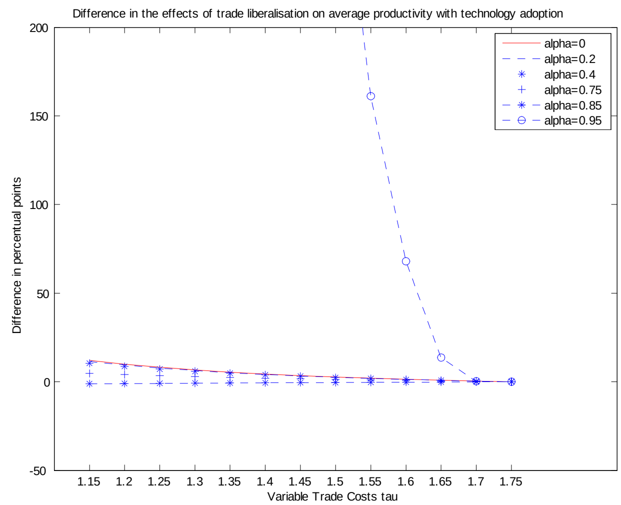


Fig 5a: The contribution of technology adoption to the effects of trade liberalisation on average productivity

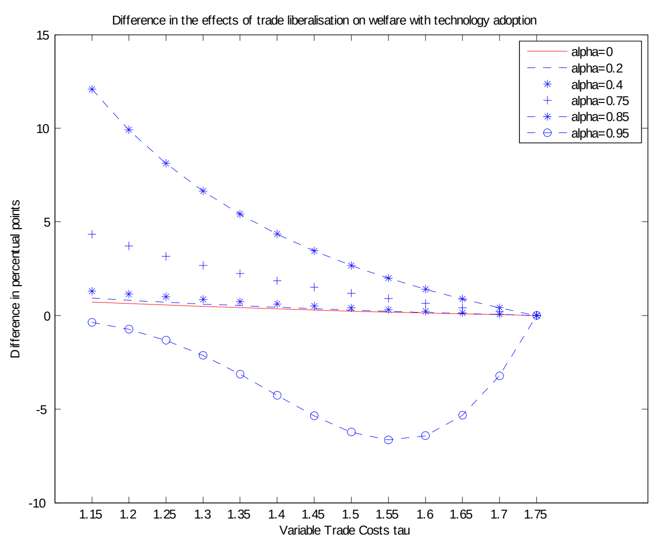


Fig 5b: The contribution of technology adoption to welfare