The macroeconomics of dollarization

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1. Introduction

Much has been written on dollarization and most contributions target, so to speak, the practitioners, addressing issues like the optimal level of official international reserves the central bank of a dollarized economy should try to hold. Still, in most cases these contributions are not related to a clear and transparent model of the economy at hand and, above all, they do not deal with the growth implications of dollarization. Dollarization is usually thought of as a short-run constraint on the possibility to implement standard stabilization macro policies, but it is not generally considered to be a constraint on longer-run growth[1]. Needless to say, this is very much in line with the idea of the irrelevance of the balance of payments in shaping the growth path of the economy[2].

On top of this, it is not always realized that dollarization as such does not change the endogenous nature of money. Money is endogenous everywhere, regardless of whether a country has its own currency or not. The banking system in a dollarized economy, as it is the case in any other economy, may and does create deposits *ex-nihilo* each time making loans to the economy is perceived as profitable. After having extended the loan, banks look for funds and, for the private banking system *as a whole* operating in an economy with its own currency, the only possible source of funding is the central bank[3]. The "central bank" of a dollarized economy is the rest of

[1] On the contrary, it is generally believed that dollarization, by stabilizing inflation at low levels, helps the economy to return to a sustained growth path.

[2] Among Keynesian people, however, the idea of an external constraint on growth has always been given a great importance – from the Harrod foreign trade multiplier to the balance of payments constrained growth models first developed by Anthony Thirlwall, from the Kaleckian (Rosa Luxemburg) idea of imperialism as a crucial tool for the survival of capitalists to the "gap models" of Hollis Chenery and his followers. On these issues, see Thirlwall (2011).

[3] For a single bank, the other sources of funds are deposits and the interbank market.

the world, since in such a case reserves are to be borrowed from abroad. As we will see, this is a crucial difference, but money is endogenous in both cases.

The aim of this paper is to fill these two gaps and build a theoretical growth model for a dollarized economy in a framework of endogenous money. We will show that, *ceteris paribus*, in a world of endogenous money the steady-state medium-term growth rate of a dollarized country is lower than that of a country with its own currency. We will also show that the negative impact on growth of whatever negative shock is magnified by dollarization. These are the two most relevant results of the paper.

What the experience of dollarized economies shows is that in times of crisis people preference for cash (the legal tender) as a store of value drastically increases (see Acosta and Guijarro, 2018, p. 236-37, for an illustration of the Ecuadorian case). Scared people just go to ATM or the bank counter and withdraw, and then hold money (cash) under the pillow or in a safe at home. The legal tender is (perceived to be) like gold – a safe asset to be held in times of troubles. Needless to say, referring to "people" or "households" is not very satisfactory, and one should incorporate a class dimension in a serious discourse on dollarization. The way rich people and the middle class react to uncertainty is not the same. The latter increases its preference for the legal tender, whereas the former strengthen their preference for holding foreign assets ("capital flights"). This is an extremely interesting and important aspect of dollarization. However, this is beyond the scope of the paper, where we intend to present a first theoretical reflection on this topic and concentrate on the growth implications of "people" having a more or less pronounced preference for cash as a store of value. For this reason, and to keep the model analytically tractable, income distribution will be treated parametrically (details below). Inflation, and therefore the dynamics of the real exchange rate, will be also treated parametrically, instead of making it endogenous and study its determinants[4]. The rationale behind these assumptions does not lie exclusively in the analytical tractability of the model. It is strictly related to our purpose. What we really want is to compare two economies with the very same essential features (same inflation and real

[4] In the post-Keynesian tradition, income distribution and inflation are strongly correlated, the latter being the outcome of the conflict over the former. Weintraub (1958) and Rowthorn (1977) are the pioneers of this "conflicting claim" view on inflation.

exchange rate, same income distribution, etc.) and this way understand the "pure" implications of having an own currency or not on the growth path of the system. In other words: imagine that dollarization has properly done its job, i.e. stabilized inflation in the dollarized country at the same (or comparable) level as that prevailing in other economies. Once this is done, what are the consequences of remaining dollarized in terms of medium-term growth and short-run fluctuations prompted by some possible shock?

2. Preference for cash

In a world of endogenous money (the world as it is), being "dollarized" or, in more general terms, not having the right to print an own currency, only constitutes a limitation to the extent that people are willing to hold some cash as a store of value. Indeed, should people be happy with their bank accounts, the limitation of sovereignty associated to dollarization would be a purely formal one: you cannot print a piece paper that people are just not willing to hold. In a cashless (better: in a framework where cash never constitutes a store of value), endogenous money world, being dollarized or not would not make any difference.

Cash (as a store of value), however, does exist. Especially in time of troubles, people go to the ATM and withdraw cash to be held in some safe at home. This is of course a perfectly rational behavior. When you are scared and think the bank system is going to collapse or simply suffer, you want cash. You want the legal tender under your pillow. All the more so when the legal tender is the US dollar, a powerful currency trusted around the world. It is then reasonable and realistic to assume that people are not happy with holding the totality of their liquid wealth in a bank deposit and prefer to keep some cash in a safe at home. Let us call α, with $0 < \alpha < 1$, the fraction of their liquid wealth households want to keep in the form of cash. Households' desire for cash materializes whenever they want. If a sufficient amount of cash is not around to satisfy households' demand, in a dollarized economy this *must* be borrowed from abroad. Indeed, printing cash is not an option (by definition of dollarization) and leaving households' demand unsatisfied neither: this would provoke the bankruptcy of the banking system and a dramatic financial mess. The very fact of wishing to hold cash (which has to be borrowed from abroad) rather than deposits (which have not) might well increase the stock of foreign debt and worsen

the current account, in a potentially destabilizing spiral we want to study and understand carefully.

It is true that, faced with the fear of some financial troubles, people increase their preference for cash everywhere, both in dollarized and non-dollarized economies. However, there is a huge difference between the two cases. In a non-dollarized economy, people know it is always possible for monetary authorities to satisfy the increased demand for cash by just printing more of it. In a dollarized economy, people know this is not possible and this is the crucial reason to have α higher than in a non-dollarized economy. For a given state of uncertainty, say, people in a dollarized economy are likely to hold a higher fraction of their wealth in cash compared to a non-dollarized economy. Hence, one way of comparing a dollarized with a non-dollarized economy is to study the implications of having α at different, exogenous levels. This is what we are going to do in the basic model presented in the paper (section 3).

One could also consider, however, what happens during a crisis (not only in response of the fear of a crisis). During a crisis people preference for cash as a store of value drastically increases. This means that α should be treated as endogenous, varying counter-cyclically. In section 4 we will then move to a more realistic framework where α will be made endogenous.

3. The basic model

Loosely speaking, the model we are going to build is Keynesian/Kaleckian. Keynesian, because the level of activity fundamentally depends on entrepreneurs expectations; Kaleckian, because at the macro level entrepreneurs end up earning what they decide to spend in the first place.

Short run

A minimalist structure to think about dollarization is illustrated in the Stock Matrix (SM) and Flow Matrix (FM) below. In order to focus on dollarization, there are no securities and households may only hold their financial wealth either in cash (the legal tender, H) or in a bank deposit, D. Without losing any generality, we assume the interest rate on deposits is zero.

SM (dollars)					
	Households	Firms	Banks	RoW	TOTAL
Deposits	D		-D		0
Cash	H			-H	0
Loans		-L	L		0
Foreign loans			$-L_f$	L_f	0
Capital		pK			pK
TOTAL	V	0	0	-NFA	V = pK + NFA

In this open economy, households' nominal wealth (V) is the sum of the value of the capital stock (pK) and the net foreign asset position (NFA) of the country. The latter, in our framework, is the difference between cash holdings (H) and net foreign debt (L_f, which may be positive or negative). So:

$$V = PK + H - L_f$$

Using the definitions v = V/PK (normalized real wealth or wealth-to-capital ratio), h = H/PK (normalized real cash in the hand of households) and $l_f = L_f/PK$ (external debt-to-capital ratio), the above identity may be written as

$$v = 1 + h - l_f \tag{1}$$

We will follow the standard practice and assume that in each moment in time (in the "short-run"), the stock of existing wealth is what it is. It is given (by the past history of the economy). Obviously, it evolves over time (in the "medium-run"). The fact that V (and then v) is given in the short-run has an important implication. The (short-run) variations of h and l_f must be the same. The economic rationale is very simple. If people go to the ATM or the bank counter and withdraw (h goes up), the only way for the banking system of a dollarized economy to satisfy this increased demand for cash is to borrow it from abroad (l_f increases by the same amount). This certainly produces real effects because the interest bill to be paid to foreigners rises, but in the short-run the economy is becoming neither richer nor poorer because of the decision of people to hold their wealth under the pillow rather than in a bank account. The same concept may be expressed

in a different way, which will reveal convenient to a deeper grasping of the dynamic analysis we will undertake at a later stage. Call α the fraction of their wealth people wish to hold in cash, i.e. H = αV. Normalized real cash in the hand of households, h, may then be expressed as h = αv. It is then possible to rewrite (1) as

$$v = \frac{1-l_f}{1-\alpha}$$

or, re-arranging

$$l_f = 1 - v(1-\alpha) \qquad \text{(1bis)}$$

This formulation will reveal useful to develop our dynamic analysis and makes it clear that in the short-run, for a given level of v, any increase in people preference for cash translates into a higher external debt-to capital ratio. To put the same thing differently: for a given level of v, a country becomes a debtor when α > (v – 1)/v.

FM (dollars)								
	Hous	Firms		Banks		RoW		TOT
		Curr	Cap	Curr	Cap	Curr	Cap	
CONS	-pC	pC						0
INV		pI	-pI					0
EXP		pX				-pX		0
IMP		-M				M		0
[memo]	Nominal GDP = pC+PI+pX –M = W + Pf + Pb + i*Lf							
Int on L		-iL		iL				0
Int on Lf				-i*Lf		i*Lf		0
Wages	W	-W						0
Profits F	Pf	-Pf						0
Profits B	Pb			-Pb				0
	Flows-of-funds							
VarCash	$-\dot{H}$						$\dot{H}$	0
VarDep	$-\dot{D}$				$\dot{D}$			0
VarL			$\dot{L}$		$-\dot{L}$			0
VarLf					$\dot{L}_f$		$-\dot{L}_f$	0
TOT	0	0	0	0	0	Sf = - NCA	$N\dot{F}A$	0

Call Y_d real GDP (expressed in units of the domestic commodity). The fundamental identity of national accounts for our open economy with no government is

$$Y_d = C + I + (X - qM) = C + I + TB$$

M represents imports expressed in units of foreign commodity, this is why they are converted into units of domestic commodity by applying the real exchange rate q, i.e. the price of the foreign commodity expressed in units of the domestic commodity[5]. In general, q = eP*/P. Here, however, e = 1 (the economy is dollarized) and we are assuming P*=1 (as is clear from table FM), so q = 1/P. The trade balance expressed in units of domestic commodity is TB = X – qM. Imports are to be thought of as domestic firms' purchases of the foreign commodity. This commodity, as it is the case for the domestically produced commodity as well, may be used for both consumption and investment purposes.

L_f is the stock of net foreign debt (or credit, when $L_f < 0$) expressed in dollars or, given that P* = 1, in units of foreign commodity. Real GNI (Y_n, expressed in units of domestic commodity) can then be defined as:

$$Y_n = Y_D - i^* q L_f = C + I + \left[X - q\left(M + i^* L_f\right)\right] = C + I + CA,$$

where CA is the real current account expressed in units of domestic commodity (NCA in table FM is the nominal current account). Domestic banks are assumed to borrow from (or lend to) the RoW at the interest rate i*. At home, they lend money at the rate i and in general i > i* (domestic banks are indeed monopolists in the domestic market[6] and apply a markup on i*). We will assume the interest rate paid to foreigners to increase with the debt-to-capital ratio: foreigners know they are lending dollars to a country unable to print them and it makes sense to think they want to get a higher interest rate (risk premium) when the debt-to-capital ratio goes up. This will allow us to discuss properly the issue of the external constraint on growth, a constraint which would

[5] This "fundamental identity" is far from being obvious. The way it is written, it implicitly assumes that "our" exports' price is P (as can be seen from table FM), which is essentially determined by internal factors. However, as stressed by Taylor (1983, p. 128), "a pure primary product exporter would have its export price determined from abroad", and therefore our model "can best be interpreted as referring to a semi-industrialized country".

[6] In this basic model, domestic banks' loans are indeed the only way for domestic firms to finance accumulation (see the tables SM and FM).

be ruled out by definition in a framework where any amount of debt could be served at an unchanged world interest rate.

We need a theory for C, one for I and one for the trade balance. In this version of the model, we do not consider the effects of income distribution on aggregate consumption (a core principle of post-Keynesian economics) and, as made explicit in tables SM and FM, we rather refer to a generic "households" earning national income (the sum of wages and non-financial firms' and banks' profits) and accumulating national wealth. This way, we may write a generic aggregate consumption function (often employed in the so-called SFC models) as

$$C = c_1 Y_n + c_2 \mathrm{qV}$$

Using the definitions of real GDP and (1), we get

$$C = c_1 \left[Y_d - i^* q L_f \right] + c_2 q V$$

In a growth model, it is convenient to normalize variables dividing by the capital stock. Using the definitions c = C/K (normalized real consumption), u = Y_d/K (output-capital ratio, used as a proxy for the degree of capacity utilization) and l_f = qL_f/K (the external debt-to-capital ratio), we may write

$$c = c_1 \left[u - i^* l_f \right] + c_2 v, \tag{2}$$

Let's move to investment. In fairly general terms and without entering here the infinite debate over the appropriate form of the investment function, one might postulate (following Joan Robinson (1962)) that non-financial firms' investments depend on their expected profit rate (r^e). Using a simple linear formulation and defining g = I/K (the accumulation rate), we have

$$g = g_0 + g_1 r^e \tag{3}$$

At any point in time, the expected profit rate is what it is and therefore is taken as given. Still, as we will see in the section of the paper devoted to dynamics, it is reasonable to assume that expected profitability increases with actual profitability (the expected profit rate is revised each time there is a gap between actual and expected profitability). The macro profit rate is the ratio between the profit bill and the value of installed capital

$$r = \frac{pC+pI+pX-M-W-iL}{PK}$$

Calling w the nominal wage, ω = w/P the real wage, N the employment level, a = N/Y_d the inverse of labor productivity (i.e. the labor-to-GDP ratio) and ψ = ωa the wage share in total GDP, one can easily calculate the profit rate as (do not forget that in our framework L = PK, look at table SM)

$$r = [(1-\psi)u - i] \tag{4}$$

The interpretation is somewhat obvious: since ψ is the share of GDP going to workers and the interest rate i measures the rent going to domestic bankers, (4) says that when more is to be left to either workers or *rentiers*, industrialists[7] get less.

As already explained, the domestic interest rate is calculated as a markup (m, taken as given) over the rate paid to foreigners and the latter, in turn, equals the interest rate prevailing in the rest of the world (i_R, taken as given; this is to be thought of as the risk-free world interest rate) plus a risk premium (σ) increasing with the external debt-to-capital ratio.

$$i = i^* + m \tag{5}$$

$$i^* = i_R + \sigma(l_f) \qquad \sigma' > 0 \tag{6}$$ [8]

Finally, a theory for the trade balance. Calling b = TB/K the normalized real trade balance, the simplest possible one is

$$b = b_0 - b_2 u \tag{7}$$

The above relation, where $b_2 > 0$, says that the trade balance worsens when GDP goes up[9]. What about the effect of a real depreciation? Here, since we are treating the real exchange rate

[7] We prefer to use the word "industrialists" (or managers) rather than "capitalists" because in our model there are no capitalists *strictu sensu*. Investments are fully financed by making recourse to bank loans and non-financial firms distribute profits to managers' households. There are no shares, and no shareholders.

[8] It could be noted that in this model a real depreciation increases l_f and then gives rise to a higher i*.

[9] For any given level of i*, q and L_f, an increase in GDP implies a higher GNI as well. Households' consumption demand goes up and as a result imports of consumption items increase. Moreover, *ceteris paribus* a higher u increases the current profit rate and then improves profit expectations, thereby stimulating firms' investment demand. This, in turn, will push imports of investment goods up.

parametrically, it is possible to think that its effect is somewhat hidden in the parameter b_0, a shift parameter that may also represent any kind of external shock (variations in the world income, in the world price of some key commodities, etc.). We do not know whether a real depreciation makes b_0 bigger or smaller – it depends on whether the Marshall-Lerner condition holds or not.

Moving from the trade balance to the current account is easy. Call ca = CA/K the normalized real current account:

$$ca = b - i^* l_f \tag{8}$$

The equilibrium in the commodity market requires

$$u = c + g + b \tag{9}$$

The system (1bis)-(9) is a complete and very simple short-run model for the determination of l_f, c, g, b, ca, r, i, i* and u. The structure of causation reveals the Keynesian/Kaleckian nature of this short-run scheme: (3) determines g in accordance with entrepreneurs' expectations (the Keynesian side), (1 bis) makes it clear that preference for cash determines l_f and (6) determines i* for that given level of the external debt-to-capital ratio; then, (5) solves for i and the sub-system (2)-(7)-(9) gives c, u and b; finally, (8) fixes ca and (4) tells entrepreneurs their profit rate. Entrepreneurs' investment expenditures are at the beginning of this causal chain, their profits at the end: at the end of each period, entrepreneurs' get what they spend at the beginning (the Kaleckian side). In this model, as in the real world, entrepreneurs are the alpha and the omega of the economy.

In order to ease following calculations, let us assume that the risk premium increases linearly with the external debt-to-capital ratio (provided this ratio is positive[10]). Formally:

$$\sigma = \gamma l_f,$$

[10] At first sight, this assumption could seem somewhat bizarre: a debtor country pays i_r plus a risk premium, but a creditor country only gets i_R. Do not forget, however, that *this is not a multi-country, but a single-country model*. It describes the case of one (dollarized) economy facing a fundamental asymmetry: it has to pay i_r plus a risk premium when indebted, but it manages to get only i_R when acting as a lender.

with

$$\gamma = \begin{cases} > 0, \ l_f > 0 \\ = 0, \ l_f \leq 0 \end{cases}$$

This way, the elasticity of the risk premium with respect to the external debt-to-capital ratio is always one. A dollarized economy is typically indebted to the rest of the world, and we will then concentrate on the case where $l_f > 0$. Given (1bis), this implies we will especially focus on the case where $v < 1/(1-\alpha)$.

The short-run solution of the system (indicated by the subscript "s"), with $l_f > 0$ ($v < 1/(1-\alpha)$) , is:

$$u_s = \frac{g_0+b_0+g_1r^e+c_2v-c_1\{i_R+\gamma[1-v(1-\alpha)]\}[1-v(1-\alpha)]}{1-c_1+b_2}$$

$$r_s = \{(1-\psi)u_s - i_R - \gamma[1-v(1-\alpha)] - m\}$$

$$g_s = g_0 + g_1r^e$$

$$b_s = b_0 - b_2u_s$$

$$c_s = c_1u_s + c_2v - c_1\{i_R + \gamma[1-v(1-\alpha)]\}[1-v(1-\alpha)]$$
$$ca_s = b_s - \{i_R + \gamma[1-v(1-\alpha)]\}[1-v(1-\alpha)]$$

For a creditor country ($v > 1/(1-\alpha)$ and then $\gamma = 0$), we would instead have

$$u_s = \frac{g_0+b_0+g_1r^e+c_2v-c_1i_R[1-v(1-\alpha)]}{1-c_1+b_2}$$

$$r_S = (1-\psi)u_s - i_R - m$$

$$c_s = c_1u_s + c_2v - c_1i_R[1-v(1-\alpha)]$$
$$ca_s = b_s - i_R[1-v(1-\alpha)]$$

The (normalized) excess demand for commodities is ed = (c + g + b – u) and short-run stability requires

$$\frac{\partial ed}{\partial u} = c_1 - b_2 - 1 < 0,$$

which is the same condition for u to be positive in equilibrium. We will assume this standard Keynesian stability (positivity) condition holds.

The impact of a higher α (and then l_f) on the level of activity is clearly negative, both in a debtor and in a creditor country:

$$\frac{\partial u_s}{\partial \alpha} = -\frac{c_1 v}{(1-c_1+b_2)}\{i_R + 2\gamma[1 - v(1-\alpha)]\} \qquad \text{DEBTOR}$$

$$\frac{\partial u_s}{\partial \alpha} = -\frac{c_1 v}{(1-c_1+b_2)} i_R \qquad \text{CREDITOR}$$

A higher α has a negative effect on GNI and then consumption spending, and this is the reason why in a demand-led model it lowers the level of activity. On top of depressing activity, a higher l_f also reduces the industrialists' profit rate, because they sell less (u_s goes down) and, in a debtor country, have to pay higher interests to domestic bankers (for a given markup, i* goes up)[11]. One could be tempted to claim that the impact on the current account is ambiguous, since the interest bill to be paid to the foreigners goes up but the trade balance improves with the contraction of the economy following a more pronounced preference for cash. It is easy to see, however, that the former effect is stronger than the latter. The relevant derivative is

$$\frac{\partial ca}{\partial \alpha} = v\{i_R + 2\gamma[1 - v(1-\alpha)]\}\left\{\frac{b_2 c_1}{(1-c_1+b_2)} - 1\right\} \qquad \text{DEBTOR}$$

$$\frac{\partial ca}{\partial \alpha} = v i_R \left\{\frac{b_2 c_1}{(1-c_1+b_2)} - 1\right\} \qquad \text{CREDITOR}$$

The term in curly brackets is certainly negative (since $(1 - c_1)(1 + b_2) > 0$) and we may safely conclude that $\partial ca/\partial \alpha < 0$: a stronger preference for cash worsens the current account.

In light of the subsequent dynamic analysis, it is also useful to see how the level of activity reacts to a higher wealth-to-capital ratio. Using the short-run solution of the model, one can see that

[11] The model is built in such a way that we cannot say that much on the impact of a real depreciation. On the one hand, a higher q increases l_f, and as we just saw this has a recessionary impact. On the other, it might increase b_0 (provided that the Marshall-Lerner condition holds), which prompts an expansionary impact. A priori, the net effect is unclear and the risk of a contractionary devaluation is always there, as first emphasized by Taylor and Krugman (1979) in their seminal paper on this topic.

$$\frac{\partial u}{\partial v} = \frac{1}{(1-c_1+b_2)}\{c_2 + (1-\alpha)c_1[i_R + 2\gamma(1 - v(1-\alpha))]\} \qquad \text{DEBTOR}$$

$$\frac{\partial u}{\partial v} = \frac{1}{(1-c_1+b_2)}\{c_2 + (1-\alpha)c_1 i_R\} \qquad \text{CREDITOR}$$

Of course, the level of activity responds positively to wealth (this is due to how the consumption function is written in the first place), and what is more interesting to note is the role played by preference for cash: in an economy with its own currency (where, admittedly, α = 0), the short-run multiplier associated to higher wealth is higher than that of a dollarized economy (α > 0).

The reaction of the current account to higher wealth (lower external debt-to-capital ratio) is instead ambiguous:

$$\frac{\partial ca}{\partial v} = (1-\alpha)[i_R + 2\gamma(1 - v(1-\alpha))]\left(1 - \frac{b_2 c_1}{(1-c_1+b_2)}\right) - \frac{b_2 c_2}{(1-c_1+b_2)} \qquad \text{DEBTOR}$$

$$\frac{\partial ca}{\partial v} = (1-\alpha) i_R \left(1 - \frac{b_2 c_1}{(1-c_1+b_2)}\right) - \frac{b_2 c_2}{(1-c_1+b_2)} \qquad \text{CREDITOR}$$

In words: a priori, we cannot say whether the improvement of the current account due to the reduction of the interest bill paid to foreigners in the case of a debtor country or, in the case of a creditor country, to the increase in the interest payments received from the foreigners (the first term on the RHS) is more or less important than the worsening of the trade balance explained by the expansion of the activity level (the second term). It might be noted that the condition for the current account to improve with a higher v is

$$i_R > \frac{b_2 c_2}{(1-\alpha)(1+b_2)(1-c_1)} - 2\gamma l_f \qquad \text{DEBTOR}$$

$$i_R > \frac{b_2 c_2}{(1-\alpha)(1+b_2)(1-c_1)} \qquad \text{CREDITOR}$$

The starting level of the wealth-to-capital ratio (and then of the external debt-to-capital ratio) and the severity with which international financial markets judge a given level of indebtness enter in a crucial way in the above condition only in case of a debtor country. This is somewhat obvious and, as we shall see, deeply affects the dynamics of the economy over time.

Something similar happens with the sensitivity of the macro profit rate to the wealth-to-capital ratio. A higher v increases the profit rate in any case, but this is especially true for an indebted

economy, since under these circumstances the reduction of the risk premium lowers the domestic interest rate as well:

$$\frac{\partial r}{\partial v} = \frac{(1-\psi)}{(1-c_1+b_2)}\{c_2 + (1-\alpha)c_1 i_R\} + \gamma(1-\alpha)\left\{1 + \frac{2(1-\psi)}{(1-c_1+b_2)}c_1[1 - v(1-\alpha)]\right\} \quad \text{DEBTOR}$$

$$\frac{\partial r}{\partial v} = \frac{(1-\psi)}{(1-c_1+b_2)}\{c_2 + (1-\alpha)c_1 i_R\} \quad \text{CREDITOR}$$

The results so far obtained are only valid in the short-run. In the model there are two state variables (v and r^e) whose value is (to be) taken as given in the short-run. Their evolution over the medium-run, however, is to be studied carefully in order to understand the dynamics of the model economy.

Medium run

Studying the evolution over time of the wealth-to-capital ratio, v, clearly requires a full understanding of the dynamics of the external debt-to-capital ratio, l_f, since foreign debt is a (negative) component of wealth.

From the definition of v, we have

$$\dot{v} = \frac{\dot{V}}{PK} - vg \tag{10}$$

Using the definition of V and the assumption H = αV, it is easy to see that

$$\frac{\dot{V}}{PK} = \frac{1}{1-\alpha}\left(g - \frac{\dot{L}_f}{PK}\right)$$

Then:

$$\dot{v} = \frac{1}{1-\alpha}\left(g - \frac{\dot{L}_f}{PK}\right) - vg \tag{11}$$

Using

$$\frac{\dot{L}_f}{PK} = \dot{l}_f + g l_f$$

one can re-express (11) as

$$\dot{v} = -\frac{1}{1-\alpha}\dot{l}_f + g\left(\frac{1-l_f}{1-\alpha} - v\right)$$

and, observing that H = αV implies $v = \left(1 - l_f\right)/(1-\alpha)$, its formulation becomes

$$\dot{v} = -\frac{1}{1-\alpha}\dot{l}_f \tag{12}$$

Expression (12) is a very useful one. On top of showing that the stationarity of the external debt-to-capital ratio implies that of the wealth-to-capital ratio (and vice versa), it says that in a dollarized economy where people hold cash as a store of value (α > 0), the relation between the variations of the external debt-to-capital ratio and those of the wealth-to-capital ratio is not one-to-one, contrary to what would happen in an economy with its own currency where people do not hold cash as a store of value (α = 0). Obviously, equation (12) also says we cannot understand the dynamics of the wealth-to-capital ratio without having first understood that of the external debt-to-capital ratio, l_f. From the definition $l_f = qL_f/K$ and remembering that q is treated parametrically (hence, $\hat{q} = 0$) we get

$$\hat{l}_f = \hat{L}_f - g \tag{13}$$

Using national accounts (the flows-of-funds in table FM are enough), one gets

$$\dot{L}_f = \dot{H} - \dot{NFA} = \dot{H} - NCA \tag{14}$$

However simple, the notion incorporated in equation (14) is key. Even if the net foreign asset position does not change (the nominal current account is zero), in a dollarized economy foreign debt increases to the extent that people want to hold more cash as a store of value.

Dividing by the stock of foreign debt and normalizing by the capital stock, (14) becomes

$$\hat{L}_f = \frac{\frac{\dot{H}}{PK} - ca}{l_f} \tag{15}$$

Let us insert (15) into (13):

$$\dot{l}_f = \frac{\dot{H}}{PK} - ca - gl_f \tag{16}$$

Now, using the assumption that people keep a fraction α of their wealth in the form of cash (H = αV), (16) may be written as

$$\dot{l}_f = \alpha \frac{\dot{V}}{PK} - ca - gl_f \tag{17}$$

By the definition of v we get

$$\frac{\dot{V}}{PK} = \dot{v} + vg$$

and using this we may write

$$\dot{l}_f = \alpha(\dot{v} + vg) - ca - gl_f \tag{18}.$$

This is not yet the end of the story. Using (1) and observing that h = αv, (18) may be written as

$$\dot{l}_f = \alpha\left(\dot{v} + \frac{1-l_f}{1-\alpha} g\right) - ca - gl_f$$

Rearranging:

$$\dot{l}_f = \alpha\dot{v} - ca + \frac{g(\alpha - l_f)}{(1-\alpha)} \tag{19}$$

Before dropping (19) into (12) to have a complete differential equation for the dynamics of the wealth-to-capital ratio, let us have a quick look to the meaning of (19) itself. To begin with, think to what happens with α = 0, i.e. in a world where people do not want to hold cash as a store of value. Equation (19) would get a much more familiar face:

$$\dot{l}_f = -(ca + gl_f)\ ,$$

meaning that in an economy with its own currency (where people are assumed not to hold cash as a store of value) the external debt-to-capital ratio diminishes with positive growth (the denominator increases) and a positive current account (the numerator goes down because less has to be borrowed). What (19) shows is that in a dollarized economy things are not that simple. There are essentially two important messages. First, the growth of (normalized) wealth as such worsens the external debt-to-capital ratio to the extent that people want to hold cash as a store of value. This happens simply because cash has to be borrowed from abroad. Second, the positive

effect of the growth of capital (and output) on the external debt-to-capital ratio is mitigated, once again, by the willingness of people to hold cash. Even more than that: if this willingness is very pronounced and/or the external debt-to-capital ratio is initially low ($\alpha > l_f$), real growth would go hand in hand with an increasing external debt-to-capital ratio (sooner or later, that growth would then become unsustainable).

Now drop (19) into (12). Using (1bis), one gets the following very simple expression for the dynamics of the wealth-to-capital ratio

$$\dot{v} = ca + g(1 - v) \tag{20}$$

Equation (20) may be usefully rewritten by specifying what ca and g depend on according to the short-run solution of the model:

$$\dot{v} = ca(v, r^e; \alpha) + g(r^e)(1 - v) = G(v, r^e; \alpha) \tag{21}$$

It is worth noting that whereas in our model the current account depends upon the cash preference parameter α (and we already saw that $\partial ca/\partial \alpha < 0$), the growth rate g only depends upon entrepreneurial expectations.

Let us then turn to the evolution of profit expectations. We will simply assume they are revised upward (downward) each time the actual profit rate is higher (lower) than expected ($\varphi > 0$):

$$\dot{r}^e = \varphi[r(v, r^e; \alpha) - r^e] = F(v, r^e; \alpha) \tag{22}$$

Expressions (21) and (22) constitute the system of two differential equations we are going to use to study the dynamics of our model economy. Before drawing the demarcation lines (isoclines) associated to the dynamic system (21)-(22) and its steady-state, let us discuss from a strictly economic standpoint the potential sources of instability operating in this model economy.

Essentially, there are two sources of instability, one internal and one external.

Internal instability is basically related to income distribution and the importance of the accelerator term of the investment function. The idea is very simple. In our model economy, higher expected profits stimulate investments, then output and sales, then actual profits. Higher actual profits, in turn, translate into higher expected profits, and this stimulates investments

again, and so on and so forth. Clearly, this is a potentially explosive dynamics. The importance of this cumulative effect depends on the slope parameter of the investment function (g_1) and on the profit share ($1 - \psi$) – in words: on how strongly investments respond to a wave of optimism and on how much of the extra-income generated by investments ends up into the hands of industrialists (those who decide investments). For the model to be dynamically stable, these two parameters cannot be too high.

[TO BE CONTINUED]

Appendix

The "Profits" curve

The equation of the "Profits" curve (22) is

$$\varphi[r(v,r^e;\alpha)-r^e]=F(v,r^e;\alpha)=0$$

This may be written as

$$F(v,r^e;\alpha)=0$$

Applying the implicit function theorem, we know that along this isocline

$$\frac{\partial v}{\partial r^e}=-\frac{F_{r^e}}{F_v}$$

Let us calculate both partial derivatives. Using the short-run solution of the model and the value of $\partial r/\partial v$ we already calculated in the text:

$$F_v=\frac{\partial \dot{r}^e}{\partial v}=\varphi\frac{\partial r}{\partial v}=$$

$$=\varphi\frac{(1-\psi)\{c_2+(1-\alpha)c_1[i_R+2\gamma[1-v(1-\alpha)]]\}+(1-\alpha)(1-c_1+b_2)\gamma}{(1-c_1+b_2)} \qquad \text{DEBTOR}$$

$$=\varphi\frac{(1-\psi)\{c_2+(1-\alpha)c_1 i_R\}}{(1-c_1+b_2)} \qquad \text{CREDITOR}$$

As to the impact of expected profitability, the relevant derivative is (for both a debtor and a creditor country)

$$F_{r^e}=\frac{\partial \dot{r}^e}{\partial r^e}=\varphi\left[\frac{\partial r}{\partial r^e}-1\right]=\varphi\left[(1-\psi)\frac{\partial u}{\partial r^e}-1\right]=\varphi\left[\frac{(1-\psi)g_1}{1-c_1+b_2}-1\right].$$

Putting things together, we may express the slope of the "Profits" curve as

$$\frac{\partial v}{\partial r^e}=-\frac{F_{r^e}}{F_v}=-\frac{[(1-\psi)g_1-(1-c_1+b_2)]}{(1-\psi)\{c_2+c_1(1-\alpha)[i_R+2\gamma[1-v(1-\alpha)]]\}+\gamma(1-\alpha)(1-c_1+b_2)} \qquad \text{DEBTOR}$$

$$\frac{\partial v}{\partial r^e}=-\frac{F_{r^e}}{F_v}=-\frac{[(1-\psi)g_1-(1-c_1+b_2)]}{(1-\psi)\{c_2+(1-\alpha)c_1 i_R\}} \qquad \text{CREDITOR}$$

The idea of preventing "internal instability" (see the discussion in the text) from materializing requires

$$(1-\psi)g_1-(1-c_1+b_2)<0$$

If we impose this condition, the "Profits" curve certainly slopes positively, regardless of whether the country is a creditor or a debtor in the international financial markets.

<u>The "Wealth" curve</u>

The equation of the "Wealth" curve (21) is

$$ca(v,r^e;\alpha)+g(r^e)(1-v)=0$$

This may be written as

$$G(r^e,v)=0$$

Applying the implicit function theorem, we know that along this isocline

$$\frac{\partial v}{\partial r^e}=-\frac{G_{r^e}}{G_v}$$

Let us calculate both partial derivatives.

$$G_v=\frac{\partial ca}{\partial v}-g$$

In the text we already calculated the value of $\partial ca/\partial v$. Using it, we get:

$$G_v=\frac{(1-c_1)(1+b_2)(1-\alpha)\{i_R+2\gamma[1-v(1-\alpha)]\}-[b_2c_2+(1-c_1+b_2)(g_0+g_1r^e)]}{(1-c_1+b_2)} \qquad \text{DEBTOR}$$

$$G_v=\frac{(1-c_1)(1+b_2)(1-\alpha)i_R-[b_2c_2+(1-c_1+b_2)(g_0+g_1r^e)]}{(1-c_1+b_2)} \qquad \text{CREDITOR}$$

Let us turn to G_{r^e} ($\partial\dot{v}/\partial r^e$):

$$G_{r^e}=\frac{\partial ca}{\partial r^e}+(1-v)g_1$$

Using the short-run solution of the model, one gets (regardless of whether a country is a creditor or a debtor in international financial markets)

$$G_{r^e}=\frac{\partial\dot{v}}{\partial r^e}=g_1\left[\frac{(1-c_1)}{(1-c_1+b_2)}-v\right]$$

Putting things together, we may express the slope of the "Wealth" curve as

$$\frac{\partial v}{\partial r^e} = -\frac{G_{r^e}}{G_v} = g_1 \frac{[v(1-c_1+b_2)-(1-c_1)]}{(1-c_1)(1+b_2)(1-\alpha)\{i_R+2\gamma[1-v(1-\alpha)]\}-[b_2c_2+(1-c_1+b_2)(g_0+g_1r^e)]} \qquad \text{DEBTOR}$$

$$\frac{\partial v}{\partial r^e} = -\frac{G_{r^e}}{G_v} = g_1 \frac{[v(1-c_1+b_2)-(1-c_1)]}{(1-c_1)(1+b_2)(1-\alpha)i_R-[b_2c_2+(1-c_1+b_2)(g_0+g_1r^e)]} \qquad \text{CREDITOR}$$

Written as they are, the two above expressions are not very useful because their RHSs depend on both v and r^e. Luckily enough, this problem may be solved by remembering that we are *on the "Wealth" curve*, meaning that r^e can be expressed as a function of v (or vice versa). This way, it will be possible to express the slope of this demarcation line as a function of v (or r^e) only. The algebraic details are tremendously boring. Take the case of a debtor country. Using the short-run solution of the model, the "Wealth" curve (ca + g(1 – v) = 0) may be written as

$$b_0 - b_2 \frac{g_0+b_0+g_1r^e+c_2v-c_1\{i_R+\gamma[1-v(1-\alpha)]\}[1-v(1-\alpha)]}{1-c_1+b_2} - \{i_R+\gamma[1-v(1-\alpha)]\}[1-v(1-\alpha)] + (g_0+g_1r^e)(1-v) = 0$$

After some calculations, one gets

$$g_1r^e = \frac{(1-c_1)(1+b_2)\{i_R+\gamma[1-v(1-\alpha)]\}[1-v(1-\alpha)]+g_0[v(1-c_1+b_2)-(1-c_1)]}{(1-c_1)-v(1-c_1+b_2)} +$$

$$+\frac{vb_2c_2-b_0(1-c_1)}{(1-c_1)-v(1-c_1+b_2)}$$

[TO BE CONTINUED]

References

[TO BE INSERTED]