Welfare-reducing licensing of a product innovation: when vertical relations matter

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ABSTRACT
This paper focuses on how the presence of an upstream monopoly supplier affects the decision of a firm on a downstream market to license its product innovation to a potential rival. Licensing can occur upon the licensee's payment of either a fixed fee or a royalty to the innovator and can lead to downstream competition *à la* Cournot or *à la* Bertrand with differentiated products. We find that licensing through a fixed fee, which arises when products are sufficiently differentiated, is never preferred to royalty licensing, irrespective of the mode of competition. This is in contrast to the case in which the upstream market is perfectly competitive. We also show that, when product differentiation is very low, the presence of upstream market power lets royalty licensing under Bertrand be welfare detrimental with respect to the hypothesis of no licensing.

JEL CLASSIFICATION: D45, L13, L41, O31

KEYWORDS: COURNOT, BERTRAND, LICENSING, PRODUCT INNOVATION, OUTSOURCING

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1 Introduction

Technology transfer is a major driver of innovative entrepreneurship and economic growth. It occurs through licensing agreements allowing a licensee to access external sources of technology and the licensor to reap a reward from an investment in R&D. There is a large body of oligopoly theory which studies the licensing arrangements involving either a firm or a research laboratory, the latter discovering a cost-reducing or a product innovation. As traditional literature shows, a licensing contract can be designed by the patentee to include an auction-based payment, a lump-sum payment, a variable royalty, or a combination of a lump-sum and a variable royalty (i.e., a two-part contract).[1]

The profit dominance of a fixed fee over a royalty contract in markets with a cost-reducing innovation by an outsider has been proved under Cournot by Kamien and Tauman (1984, 1986) and Kamien et al. (1992), the latter addressing both the quantity and the price competition case.[2] The opposite result, with a per unit royalty being more profitable than a fixed fee, has been achieved by Sen (2005) in a Cournot oligopoly with the number of licenses subject to an integer constraint and by Muto (1993) in a context *à la* Bertrand. By addressing the case of a patentee that is also a producer in a Cournot market, Wang (1998), Wang (2002) and Kamien and Tauman (2002) find that a royalty contract can be more advantageous for the patent holder, but not for consumers that are better off under fixed fee licensing. Moreover, the dominance of a royalty scheme over a fixed fee scheme has been shown to be robust to Bertrand competition by Wang and Yang (1999) and Colombo (2015).

The optimality of a licensing scheme over another, largely debated in process innovation literature, has been considered in a product innovation scenario with an external innovator by Bagchi and Mukherjee (2014). The latter compare a royalty scheme with an auction scheme (i.e., a fixed fee) under either Cournot or Bertrand competition, finding that both the innovator and the society can be better off adopting royalty licensing when the number of potential licensees is sufficiently large, regardless of the mode of competition. A comparison between royalty and fixed fee licensing with an external innovator is developed also in Li and Wang (2010) who assume that the patent improves the quality of an existing product and prove that the preference of a royalty with respect to a fixed fee contract is observed when the innovation is low enough. Focusing on a vertically differentiated duopolistic model, Li and Song (2008) study the optimal contract of a high-quality producer that can license a new technology or an obsolescent technology to a low-quality producer by means of a royalty or a fixed fee: they prove that the patentee prefers to licence the new technology, adopting a royalty licensing contract.

[1] Kamien and Tauman (1984), Faulì-Oller and Sandonis (2002), Faulì-Oller et al. (2013), Colombo and Filippini (2015), Chang et al. (2017), among others, analyze the patentee's optimal behavior, and the associated welfare implications, when licensing occurs *via* a combination of a fixed payment and a per unit royalty, under either product or process innovation.

[2] Kamien and Tauman (1986) and Kamien et al. (1992) have also demonstrated the welfare dominance of a fixed fee over a royalty.

All the above works assume either independent firms or firms procuring inputs from perfectly competitive markets. In this paper, however, we aim at exploring the optimal licensing behavior of a product innovator who is an incumbent retailer in a market in which a crucial input is supplied by an upstream monopolist.[3] Within the literature on patent licensing in vertical markets, the present work is most closely related to Arya and Mittendorf (2006). The latter addresses the question of whether fixed fees or royalties should be included in a contract allowing a patentee to license a new product innovation. Through successful licensing, the patentee competes *à la* Cournot on the downstream market against the licensee, while they both outsource a crucial input to an upstream monopolist which sets discriminatory prices for the input. In this work licensing turns out to occur only by means of a royalty contract which puts the downstream rival in a weaker position and contributes to procure lower supplier prices. By improving vertical pricing efficiency, licensing also improves social welfare. The same mechanism is at work in the Cournot model of Chang et al. (2013) who compare a royalty with a fixed fee contract assuming that an outside innovator licenses her cost-reducing technology to one or both downstream firms, obtaining the opposite result with respect to that of Kamien and Tauman (1986). Indeed, royalty turns out to be the optimal licensing strategy since it exerts a downward pressure on the wholesale prices, thus mitigating the effect of double marginalization and allowing the patentee to benefit from this input cost advantage. Chang et al. (2013) also find a negative impact of royalty licensing on social welfare with respect to no licensing when the innovation size is small. Finally, a comparison between a fixed fee and a per unit royalty contract is developed in a unionized labor market by Mukherjee (2010) who assumes that the wage rates are determined through a Nash bargaining mechanism between firms and labor unions. He shows that, under a sufficiently high bargaining power of the labor union, an external innovator prefers to license her innovation through a royalty rather than a fixed-fee or an auction, since the royalty method allows her to get lower wage rates.

The present work extends to the presence of both differentiated products and price competition the model that Arya and Mittendorf (2006) develop in a Cournot framework with homogeneous products. As in Arya and Mittendorf (2006), we study the patentee's optimal choice to license her innovation when two types of contracts are available to her, a fixed fee and a per unit royalty contract, also assessing the impact of the licensing choices on social welfare. We show that, regardless of the mode of competition, a royalty is always advantageous for the patentee with respect to no licensing, while licensing through a fixed fee occurs when highly differentiated markets allow the patentee to benefit from licensing revenues, keeping her own market profits relatively high. We

[3] Input market power is observed in different industries, as the energy sector, the market of microprocessors, aircraft-engines. The relevance of the input market is linked to the observation that in a lot of cases, firms prefer outsourcing to obtain a necessary input, instead of in-house production, not only if they can buy the input at a lower price than their internal production cost, but in same cases also if in-house production cost is lower, as for example shown in Arya and al. (2008) and Kabiraj and Sinha (2014, 2016).

thus demonstrate that the Arya and Mittendorf (2006)'s result that the patentee prefers not to license through a fixed fee is conditioned on the assumption of homogeneous products. Moreover, a comparison between the two types of contracts reveals that the patentee is better off by licensing through a per unit royalty than a fixed fee regardless of the degree of product substitutability and the type of market competition, due to the strategic gains she gets from profitably orienting the input price's choice by the upstream supplier. These profit gains, however, lead the society to be worse off than under fixed fee, which may be in contrast with the case of perfectly competitive upstream market. In the latter, indeed, fixed fee turns out to be more profitable than royalty licensing when a sufficiently high degree of product differentiation lets the patentee benefit from significant licensing revenues without putting the rival at a cost disadvantage, also keeping profits on her own market relatively high.[4] Finally, we find that royalty licensing in a Bertrand setting may be welfare-detrimental with respect to no licensing, as long as products are very close substitutes. In such circumstances, indeed, royalty licensing succeeds to soften downstream market competition by letting the upstream input prices be relatively high, thus enhancing the double marginalization problem along the vertical chain and hurting vertical efficiency as compared to a monopoly setting.[5]

The contribution of this work to the existing literature on patent licensing is twofold. On the one hand, it highlights the role of product differentiation in letting fixed fee licensing emerge at equilibrium in a vertical market, also showing the circumstances under which a royalty contract yields higher profits but lower welfare than a fixed fee contract. On the other hand, this study demonstrates that the introduction of market competition through royalty licensing may not improve vertical efficiency and social welfare when firms compete in prices on the downstream market.

The paper is organized as follows. The model is examined in Section 2, which includes the analysis of both the benchmark case with a perfectly competitive input market and the case with a monopoly input market. In Section 3 we extend the model to the assumption that the monopolist licenses one of her variety. Finally, Section 4 summarizes the results and concludes.

2 The model

Firm 1 has a patent for a technology that gives it exclusive rights to produce a final product.[6] It can decide to license its technology to a potential rival, firm

[4] The choice of a fixed fee contract keeps being the welfare optimal strategy under a perfectly competitive upstream market.

[5] This result resembles that found by Faulì-Oller and Sandonis (2002) in a Bertrand framework with process innovation and independent firms, where it is shown to be conditioned on the presence of sufficiently large (non-drastic) innovation and very high product substitutability.

[6] We assume that the patentee produces only a variety of the good, for example because the rival has a lower marketing or development cost that allows her to produce another variety. Moreover, as specified by Bagchi and Mukherjee (2014), product differentiation can be created

2, through a per unit royalty r or a fixed fee F licensing contract. In the case of licensing, firm 2 enters the final product market and downstream competitors face the following direct demand functions, a simplified version of Singh and Vives (1984):

$$q_1 = \frac{a(1-\gamma) - p_1 + \gamma p_2}{(1-\gamma^2)} \tag{1}$$

$$q_2 = \frac{a(1-\gamma) - p_2 + \gamma p_1}{(1-\gamma^2)} \tag{2}$$

or the inverse demand functions:

$$p_1 = a - q_1 - \gamma q_2 \tag{3}$$

$$p_2 = a - q_2 - \gamma q_1 \tag{4}$$

where p_1 and p_2 are respectively firm 1 and firm 2's prices, q_1 and q_2 their outputs and $a > 0$. The parameter $\gamma \in [0,1]$ measures the degree of product substitutability: if $\gamma = 0$ goods are independent; while if $\gamma = 1$ they are homogeneous. We assume that one unit of the final product is produced with one unit of the input and the cost to convert the input into the final product is normalized to zero, with zero fixed costs. The cost to produce the input is c, where $c < a$.

2.1 Upstream perfectly competitive market

In this first part we assume perfect competition on the input market. At the first stage firm 1 chooses to license or not to license its patent and firm 2 decides whether to accept the patent-holder's offer. At the last stage of the game, firms engage in downstream market competition *à la* Cournot or *à la* Bertrand with differentiated products.

The subgame Nash equilibrium is found by backward induction, solving the following subgames:

- no licensing: firm 1 does not license the patent and preserves its monopolistic power on downstream market. Its profit can be expressed as follows:

$$\pi_1 = (p_1 - c)q_1 \tag{5}$$

- licensing with a fixed fee: firm 1 receives a fixed amount of money F by firm 2, irrespective of the licensee's production. Firms' profits are:

$$\pi_1 = (p_1 - c)\, q_1 + F \tag{6}$$

$$\pi_2 = (p_2 - c)\, q_2 - F \tag{7}$$

by consumer perception for different brand names, packaging, after sales services, not for the production technologies. This assumption is a standard in the literature on product innovation i.e. in Chang et al. (2017), Kitagawa et al. (2014) and Kitagawa et al. (2018). Also San Martin and Saracho (2016), by using Kitagawa et al. (2014)'s model, assume that the monopolist produces a single variety of the good.

- licensing with a royalty: firm 1 licenses its technology receiving a royalty r for each unit produced by firm 2. Firms' profits can be written as follows:

$$\pi_1 = (p_1 - c)q_1 + rq_2 \tag{8}$$

$$\pi_2 = (p_2 - c)q_2 - rq_2 \tag{9}$$

In this section we analyze the different situations when firm 1 is monopolist on the downstream market and when it licenses its technology to firm 2, by means of a fixed amount F or through a per unit royalty r in the two different cases of quantity and price competition.

2.1.1 The no licensing case

By maximizing the firm 1's profit in (5), we obtain:

$$q_1^M = \frac{(a-c)}{2}$$

$$p_1^M = \frac{(a+c)}{2}$$

We can write firm 1's profit, consumer surplus and social welfare as follows:

$$\pi_1^M = \frac{(a-c)^2}{4} \tag{10}$$

$$S^M = \frac{(a-c)^2}{8} \tag{11}$$

$$W^M = \frac{3(a-c)^2}{8} \tag{12}$$

2.1.2 Licensing under Cournot competition

In this subgame, we assume that firms play *à la* Cournot facing the inverse demands functions (3) and (4). In the following sections, we analyze the case of licensing by means of a fixed payment and a per unit royalty.

Licensing through a fixed fee At the retail market stage, the maximization of profits in (6) and (7) with respect to q_1 and q_2 yields the following reaction functions, respectively for firm 1 and firm 2:

$$q_1 = \frac{a - \gamma q_2 - c}{2}$$

$$q_2 = \frac{a - \gamma q_1 - c}{2}$$

which exhibit strategic substitutability. By solving the system of the two reaction functions, we obtain the optimal quantities:

$$
\begin{aligned}
q_1^F &= \frac{a-c}{2+\gamma} \\
q_2^F &= \frac{a-c}{2+\gamma}
\end{aligned}
$$

By substituting the optimal quantities in (6) and (7), we obtain the following firms' profits:

$$
\begin{aligned}
\pi_1^F &= \frac{(a-c)^2}{(2+\gamma)^2} + F \\
\pi_2^F &= \frac{(a-c)^2}{(2+\gamma)^2} - F
\end{aligned}
$$

At the licensing stage, the rival accepts the firm 1's offer if its profit is greater or equal to that of no licensing. By considering that in absence of licensing firm 2 does not compete on the product market, the fixed amount paid by the entrant must respect the following condition:

$$
F \leq \frac{(a-c)^2}{(2+\gamma)^2}
$$

The maximum amount that firm 1 can charge is $F = \frac{(a-c)^2}{(2+\gamma)^2}$. By substituting it in the previous profit equations, we obtain the licensor's profit as follows:

$$
\pi_1^F = \frac{2(a-c)^2}{(2+\gamma)^2} \tag{13}
$$

while $\pi_2^F = 0$.

Consumer surplus and social welfare are:

$$
S^F = \frac{(a-c)^2(1+\gamma)}{(2+\gamma)^2} \tag{14}
$$

$$
W^F = \frac{(a-c)^2(3+\gamma)}{(2+\gamma)^2} \tag{15}
$$

By comparing firm 1's profit in the case of fixed fee licensing in (13) with the monopolistic profit in (10), consumer surplus in (14) and social welfare in (15) respectively with those of monopoly (11) and (12), we obtain the following differentials:

$$
\begin{aligned}
\pi_1^F - \pi_1^M &= \tfrac{(a-c)^2(4-\gamma^2-4\gamma)}{4(2+\gamma)^2} \geq 0 \text{ if } \gamma \leq 0.82843 \\
S^F - S^M &= \tfrac{(a-c)^2(4-\gamma^2-4\gamma)}{8(2+\gamma)^2} \geq 0 \\
W^F - W^M &= \tfrac{(a-c)^2(12-4\gamma-3\gamma^2)}{8(2+\gamma)^2} \geq 0
\end{aligned}
$$

and we state the following remark:

Remark 1 *If firms engage in quantity competition, from the perspective of the patentee licensing with a fixed fee is profitable as long as* $\gamma \leq 0.82843$. *This is a welfare-improving strategy with respect to no licensing.*

Licensing with a per unit royalty By maximizing firms' profits in (8) and (9) with respect to q_1 and q_2, we obtain the following reaction functions:

$$\begin{aligned} q_1 &= \frac{a - \gamma q_2 - c}{2} \\ q_2 &= \frac{a - \gamma q_1 - c - r}{2} \end{aligned}$$

which exhibit strategic substitutability. Notice that firm 1 maximizes its profit without considering the effect of its aggressiveness on the rival's output through the royalty r.

By solving the system above, we obtain the optimal quantities as functions of the royalty r:

$$\begin{aligned} q_1 &= \frac{(a-c)(2-\gamma) + \gamma r}{4-\gamma^2} \\ q_2 &= \frac{(a-c)(2-\gamma) - 2r}{4-\gamma^2} \end{aligned}$$

Notice that if the patentee charges a higher value of r, the patentee's production increases and, for strategic substitutability, the licensee's output decreases.

At the licensing stage, the maximization of firm 1's profit in (8), incorporating the optimal quantities, gives the equilibrium royalty rate:

$$r^* = \frac{(a-c)(8+\gamma^3-4\gamma^2)}{2(8-3\gamma^2)}$$

In the following Figure 1 we observe that, as long as goods are sufficiently differentiated, the patentee sets a value of the royalty rate that is decreasing in γ. This pattern is motivated by the aim of the patentee to benefit from licensing revenues by keeping q_2 relatively high, i.e., by reducing the rival's cost disadvantage, since sufficiently low γ ensure that q_1 does not decrease consistently following an increase of q_2. Conversely, when goods are less differentiated, namely the downstream market is more competitive, the patentee charges a royalty rate that increases in γ, which allows her to mostly benefit from profits on her own channel by exploiting a competitive advantage against a high-cost rival.

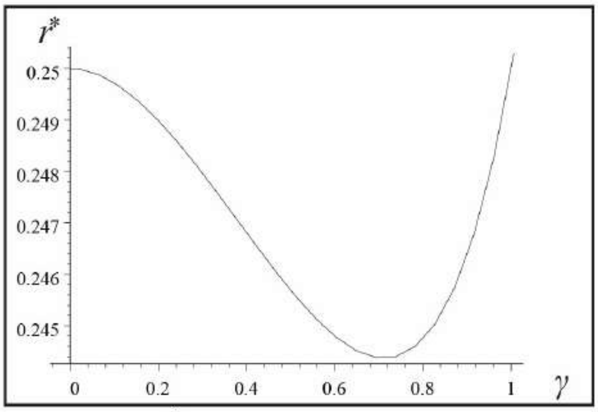

Fig. 1: The trend of r^* in case of upstream perfect market competition.

The equilibrium quantities are:

$$
\begin{aligned}
q_1^R &= \frac{(a-c)(2-\gamma)(4+\gamma)}{2(8-3\gamma^2)} \\
q_2^R &= \frac{2(a-c)(1-\gamma)}{(8-3\gamma^2)}
\end{aligned}
$$

Firms' profits are:

$$\pi_1^R = \frac{(a-c)^2(2-\gamma)(6-\gamma)}{4(8-3\gamma^2)} \tag{16}$$

$$\pi_2^R = \frac{4(a-c)^2(1-\gamma)^2}{(8-3\gamma^2)^2} \tag{17}$$

We obtain the following consumer surplus and social welfare:

$$S^R = \frac{(a-c)^2(80+9\gamma^4+12\gamma^3-76\gamma^2)}{8\,(8-3\gamma^2)^2} \tag{18}$$

$$W^R = \frac{(a-c)^2(304+3\gamma^4+60\gamma^3-100\gamma^2-192\gamma)}{8\,(8-3\gamma^2)^2} \tag{19}$$

By comparing firm 1's profit in (16) with that of monopoly in (10), consumer surplus in (18) and social welfare in (19) with those in absence of licensing, we analyze the following differentials:

$$
\begin{aligned}
\pi_1^R - \pi_1^M &= \tfrac{(a-c)^2(1+\gamma^2-2\gamma)}{8-3\gamma^2} \geq 0 \\
S^R - S^M &= \tfrac{(a-c)^2(4+3\gamma^3-7\gamma^2)}{2(8-3\gamma^2)^2} \geq 0 \\
W^R - W^M &= \tfrac{(a-c)^2(28-6\gamma^4+15\gamma^3+11\gamma^2-48\gamma)}{2(8-3\gamma^2)^2} \geq 0
\end{aligned}
$$

We can introduce the following remark:

Remark 2 *By assuming royalty licensing, if firms compete à la Cournot, firm 1 gains more by licensing its innovation to the potential rival regardless of the product differentiation parameter. This is a welfare improving policy with respect to no licensing.*

Fixed fee vs. royalty Here we compare the two types of licensing contracts and we obtain the following differentials:

$$\begin{aligned}
\pi_1^F - \pi_1^R &= \tfrac{(a-c)^2(16-\gamma^4+4\gamma^3-8\gamma^2-16\gamma)}{4(2+\gamma)^2(8-3\gamma^2)} \geq 0 \text{ if } \gamma \leq 0.787\,83 \\
S_1^F - S_1^R &= \tfrac{(a-c)^2(192-9\gamma^6+24\gamma^5+64\gamma^4-128\gamma^3-160\gamma^2+192\gamma)}{8(8-3\gamma^2)^2(2+\gamma)^2} \geq 0 \\
W_1^F - W_1^R &= \tfrac{(a-c)^2(320-3\gamma^6+64\gamma^4-32\gamma^3-288\gamma^2+64\gamma)}{8(8-3\gamma^2)^2(2+\gamma)^2} \geq 0
\end{aligned}$$

We can summer the previous analyses in the following proposition:

Proposition 1 *Under quantity competition in the interval* $\gamma \leq 0.82843$ *licensing with a fixed fee is preferred to a royalty contract as long as* $\gamma \leq 0.787\,83$, *otherwise a variable payment is the optimal licensing contract for the patentee. Society is better off by licensing through a fixed fee.*

Proof. It follows from a) a comparison between the patentee's profit in the case of fixed fee licensing in (13) with that under per unit royalty licensing in (16); b) a comparison of social welfare in (15) with that in (19).

2.1.3 Licensing under Bertrand competition

In this subgame we assume that firms play *à la* Bertrand facing the direct demand functions (1) and (2).

Licensing through a fixed fee By maximizing their profit functions in (6) and (7) with respect to prices, we obtain the following reaction functions:

$$\begin{aligned}
p_1 &= \frac{a(1-\gamma)+\gamma p_2 + c}{2} \\
p_2 &= \frac{a(1-\gamma)+\gamma p_1 + c}{2}
\end{aligned}$$

which exhibit strategic complementarity. By solving the system, the equilibrium prices are:

$$
\begin{aligned}
p_1^F &= \frac{a(1-\gamma)+c}{2-\gamma} \\
p_2^F &= \frac{a(1-\gamma)+c}{2-\gamma}
\end{aligned}
$$

By using these results, we can write firms' profits as follows:

$$
\begin{aligned}
\pi_1^F &= \frac{(a-c)^2(1-\gamma)}{(2-\gamma)^2(1+\gamma)} + F \\
\pi_2^F &= \frac{(a-c)^2(1-\gamma)}{(2-\gamma)^2(1+\gamma)} - F
\end{aligned}
$$

The maximum value of F that firm 2 accept to pay is:

$$
F = \frac{(a-c)^2(1-\gamma)}{(2-\gamma)^2(1+\gamma)}
$$

By substituting it in the previous profit equations, we obtain the following licensor's profit:

$$
\pi_1^F = \frac{2(a-c)^2(1-\gamma)}{(2-\gamma)^2(1+\gamma)} \tag{20}
$$

while $\pi_2^F = 0$.
Consumer surplus and social welfare are:

$$
S^F = \frac{(a-c)^2}{(2-\gamma)^2(1+\gamma)} \tag{21}
$$

$$
W^F = \frac{(a-c)^2(3-2\gamma)}{(2-\gamma)^2(1+\gamma)} \tag{22}
$$

We consider the following differentials by comparing the firm 1's profit in case of licensing (20) with the monopolistic profit in (10), consumer surplus in (21) and social welfare in (22) with those of monopoly (11) and (12):

$$
\begin{aligned}
\pi_1^F - \pi_1^M &= \tfrac{(a-c)^2(4-\gamma^3+3\gamma^2-8\gamma)}{4(1+\gamma)(2-\gamma)^2} \geq 0 \text{ if } \gamma \leq 0.611\,71 \\
S^F - S^M &= \tfrac{(a-c)^2(4-\gamma^3+3\gamma^2)}{8(1+\gamma)(2-\gamma)^2} \geq 0 \\
W^F - W^M &= \tfrac{(a-c)^2(12-3\gamma^3+9\gamma^2-16\gamma)}{8(1+\gamma)(2-\gamma)^2} \geq 0
\end{aligned}
$$

We get the results summarized in the following remark:

Remark 3 *If firms compete à la Bertrand, the innovator decides to licence her technology with a fixed payment as long as* $\gamma \leq 0.611$. *This is a welfare improving choice with respect to no licensing.*

We observe that if products are sufficiently differentiated, for the innovator it is profitable to license by means of a fixed payment regardless of the mode of market competition, consistently with Wang (2002) and Wang and Yang (1999)'s results in a process innovation framework under quantity and price competition, respectively. If markets are fairly unrelated for the high degree of product differentiation, licensing with a fixed fee allows the patentee to benefit from significant licensing revenues, keeping her market profit relatively high. Instead when goods are more similar, the patentee preserves her monopolistic power because in the case of licensing, the intense market competition would cause a collapse of her total profit. For the presence of a rival on the market consumers are better off and a consequent improvement of social welfare is observed.

Licensing with a per unit royalty At the market stage, firms play *à la* Bertrand facing the direct demand functions (1) and (2). The maximization of firms' profits (8) and (9) with respect to prices yields the following reaction functions for firm 1 and firm 2, respectively:

$$\begin{aligned} p_1 &= \frac{a(1-\gamma)+\gamma(p_2+r)+c}{2} \\ p_2 &= \frac{a(1-\gamma)+\gamma p_1+c+r}{2} \end{aligned}$$

which exhibit strategic complementary. Notice that the licensor is induced to behave in a less aggressive way: by setting a higher price, she raises both firm 2's output and her licensing revenues. By solving the system, we obtain the optimal prices as functions of r:

$$\begin{aligned} p_1 &= \frac{(2+\gamma)(a(1-\gamma)+c)+3r\gamma}{4-\gamma^2} \\ p_2 &= \frac{(2+\gamma)(a(1-\gamma)+c)+r(2+\gamma^2)}{4-\gamma^2} \end{aligned}$$

We observe that if the patentee sets a higher royalty, market prices increase.

At the previous stage of the game, the maximization of firm 1's profit (8), calculated at the optimal prices, gives the following equilibrium royalty rate:

$$r^* = \frac{(a-c)(2+\gamma)(4+\gamma^2-2\gamma)}{2(8+\gamma^2)}$$

which shows the same non-monotone pattern observed in the quantity competition case (Figure 1). Indeed, as long as products are sufficiently differentiated, the patentee sets a royalty rate that decreases in γ, with the aim to reduce the cost differences with the rival and benefit from higher profits on both channels. By contrast, when products are more substitutes, r^* increases in γ, thus letting the royalty act as a commitment to soften harsher market competition through higher prices.

We thus obtain the following equilibrium prices:

$$\begin{aligned} p_1^R &= \frac{a(8-\gamma^2+2\gamma)+c(8+3\gamma^2-2\gamma)}{2(8+\gamma^2)} \\ p_2^R &= \frac{a(12-\gamma^3+2\gamma^2-4\gamma)+c(4+\gamma^3+4\gamma)}{2(8+\gamma^2)} \end{aligned}$$

We obtain firms' profits:

$$\pi_1^R = \frac{(a-c)^2(12+\gamma^3+\gamma^2+4\gamma)}{4\,(8+\gamma^2)\,(1+\gamma)} \tag{23}$$

$$\pi_2^R = \frac{(a-c)^2(2+\gamma^2)^2(1-\gamma)}{(8+\gamma^2)^2\,(1+\gamma)} \tag{24}$$

Consumer surplus and social welfare are:

$$S^R = \frac{(a-c)^2(80+5\gamma^5+\gamma^4+24\gamma^3+36\gamma^2+16\gamma)}{8\,(8+\gamma^2)^2\,(\gamma+1)} \tag{25}$$

$$W^R = \frac{(a-c)^2(304-\gamma^5+11\gamma^4+16\gamma^3+108\gamma^2+48\gamma)}{8\,(8+\gamma^2)^2\,(\gamma+1)} \tag{26}$$

If we compare firm 1's profit in (23) with that of monopoly in (10), consumer surplus in (25) and social welfare in (26) with those observed in case of no licensing (11) and (12), we obtain the following differentials:

$$\begin{aligned} \pi_1^R-\pi_1^M &= \tfrac{(a-c)^2(1-\gamma)}{(1+\gamma)(8+\gamma^2)} \geq 0 \\ S^R-S^M &= \tfrac{(a-c)^2(4+\gamma^5+2\gamma^3+5\gamma^2-12\gamma)}{2(1+\gamma)(8+\gamma^2)^2} \geq 0 \text{ if } \gamma \leq 0.420\,49 \\ W^R-W^M &= \tfrac{(a-c)^2(28-\gamma^5+2\gamma^4-8\gamma^3+15\gamma^2-36\gamma)}{2(1+\gamma)(8+\gamma^2)^2} \geq 0 \end{aligned}$$

We can introduce the following remark:

Remark 4 *If firms play à la Bertrand, royalty licensing is always profitable for the innovator. This is a welfare-improving strategy with respect to no licensing, regardless of the product differentiation parameter.*

Consistently with results spread in literature on process innovation, we observe that the innovator always licenses her technology with a per unit royalty contract, regardless of the degree of product differentiation and the mode of market competition.[7] Indeed, royalty licensing leads the patentee to benefit from a higher market advantage caused by a higher cost imposed to the rival. By assessing the welfare implications, we observe that in the case of quantity competition consumers are always better off under royalty licensing with respect to no licensing, while under price competition high prices, due to the commitment effect of the royalty, reduce consumer surplus without overall hurting social welfare with respect to no licensing.

Fixed fee vs. royalty By comparing the two licensing contracts and the welfare implications, we obtain the following differentials:

$$
\begin{aligned}
\pi_1^F - \pi_1^R &= \frac{(a-c)^2(16-\gamma^5+3\gamma^4-12\gamma^3+8\gamma^2-32\gamma)}{4(8+\gamma^2)(1+\gamma)(2-\gamma)^2} \text{ if } \gamma \leq 0.520\,54 \\
S_1^F - S_1^R &= \frac{(a-c)^2\left(192-5\gamma^6+24\gamma^5-64\gamma^4+128\gamma^3-96\gamma^2+64\gamma\right)}{8(2-\gamma)^2(8+\gamma^2)^2} \geq 0 \\
W_1^F - W_1^R &= \frac{(a-c)^2\left(320+\gamma^6-16\gamma^5+32\gamma^4-96\gamma^3+160\gamma^2-320\gamma\right)}{8(2-\gamma)^2(8+\gamma^2)^2} \geq 0
\end{aligned}
$$

We summarize the above results in the following proposition.

Proposition 2 *If firms face price competition in the interval $\gamma \leq 0.61171$ licensing by means of a fixed fee is more advantageous with respect to royalty licensing as long as $\gamma \leq 0.520\,54$, otherwise a royalty contract is preferred. Society is better off under a fixed fee with respect to a royalty contract.*

Proof. It follows from a) a comparison between the patentee's profits in (20) with those in (23); b) a comparison of social welfare in (22) with that in (26).

We observe that licensing with a fixed payment is more profitable than a royalty contract as long as products are fairly differentiated. The low market competition allows the patentee to benefit from sufficiently high licensing revenues without putting the licensee in a weaker position for a cost disadvantage and also preserving a relatively high market profit. This optimal licensing contract, by keeping a cost symmetry, leads the licensee to produce more than that under royalty licensing, thus enhancing the total output and consumer surplus. A higher degree of product substitutability pushes the patentee to prefer licensing with a royalty that negatively impacts on social welfare. This result is consistent with Wang (2002) who shows in a process innovation scenario that, for certain conditions regarding the product differentiation parameter and the technological gap, royalty is more advantageous for the patentee, but not for consumers that are better off under fixed fee.

[7] If the analysis is replicated infroducing also a fixed amout F, it is observed that licensing with a two-part tariff is advantageous for the monopolist and for society regardless of the product differentiation parameter and the mode of competition.

2.2 The upstream monopoly case

In this subsection we assume that the upstream market is monopolized by an input supplier, firm S, that applies two different prices t_1 and t_2 for the input, respectively to firm 1 and firm 2.

The upstream supplier's profit can be written as follows:

$$\pi_S = (t_1 - c)\, q_1 + (t_2 - c)\, q_2 \tag{27}$$

We solve a licensing game of three stages: at the first stage, the innovator decides to license or not to license her patent through a fixed fee or a royalty and firm 2 decides whether to accept the patent-holder's offer. At the second stage, the supplier sets discriminatory prices for the input, t_1 to firm 1 and t_2 to firm 2. At the last stage of the game, firms compete on the market facing a quantity or price competition with differentiated products.

The subgame perfect Nash equilibrium of this game is found by backward induction, solving the following subgames:

- no licensing: firm 1 does not license the patent. Its profit can be expressed as follows:

$$\pi_1 = (p_1 - t_1) q_1 \tag{28}$$

- licensing with a fixed fee: firm 1 receives a fixed amount of money F by firm 2 that does not depend on the licensee's production. Firms' profits are:

$$\pi_1 = (p_1 - t_1)\, q_1 + F \tag{29}$$
$$\pi_2 = (p_2 - t_2)\, q_2 - F \tag{30}$$

- licensing with a royalty: firm 1 licenses its technology receiving a variable payment r for each unit produced by firm 2. Firms' profits can be written as follows:

$$\pi_1 = (p_1 - t_1) q_1 + r q_2 \tag{31}$$
$$\pi_2 = (p_2 - t_2) q_2 - r q_2 \tag{32}$$

In this section we develop the different situations when firm 1 is monopolist on downstream market and when it licenses its technology to firm 2, by means of a fixed amount F or through a per unit royalty r in the two different cases of quantity and price competition.

2.2.1 The no licensing case

The innovator maximizes her profit (28) and obtains the optimal quantity:

$$q_1 = \frac{a - t_1}{2}$$

Incorporating this optimal quantity, the upstream monopolist maximizes its profit with respect to t_1. The equilibrium input price is:

$$t_1^M = \frac{a+c}{2}$$

The optimal quantity and price are:

$$\begin{aligned} q_1^M &= \frac{a-c}{4} \\ p_1^M &= \frac{3a+c}{4} \end{aligned}$$

We obtain the following firms' profits:

$$\pi_1^M = \frac{(a-c)^2}{16} \tag{33}$$

$$\pi_S^M = \frac{(a-c)^2}{8} \tag{34}$$

Consumer surplus and social welfare are:

$$S^M = \frac{1}{32}(a-c)^2 \tag{35}$$

$$W^M = \frac{7}{32}(a-c)^2 \tag{36}$$

2.2.2 Licensing under Cournot competition

We assume that at the downstream competition stage firms play *à la* Cournot facing the inverse demand functions (3) and (4).

Licensing through a fixed fee The maximization of profits (29) and (30) with respect to quantities yields the following reaction functions:

$$\begin{aligned} q_1 &= \frac{a-\gamma q_2 - t_1}{2} \\ q_2 &= \frac{a-\gamma q_1 - t_2}{2} \end{aligned}$$

which exhibit strategic substitutability. Notice that a reduction of the input price charged to a firm, shifts out its reaction function increasing its output. By solving the system of the two reaction functions, we obtain the optimal quantities as functions of t_1 and t_2:

$$
\begin{aligned}
q_1 &= \frac{a(2-\gamma)+\gamma t_2-2t_1}{4-\gamma^2} \\
q_2 &= \frac{a(2-\gamma)+\gamma t_1-2t_2}{4-\gamma^2}
\end{aligned}
$$

The quantity produced by each firm decreases in its own input price and increases in the rival's input price.

At the second stage of the game, firm S maximizes its profit in (27) with respect to t_1 and t_2, taking into account the optimal quantities. The equilibrium prices are:

$$
\begin{aligned}
t_1^F &= \frac{a+c}{2} \\
t_2^F &= \frac{a+c}{2}
\end{aligned}
$$

It is easy to observe that in this case the upstream monopolist charges the same input price to both downstream firms, equal to that set in the case of downstream monopoly.

The equilibrium quantities are:

$$
\begin{aligned}
q_1^F &= \frac{(a-c)}{2(2+\gamma)} \\
q_2^F &= \frac{(a-c)}{2(2+\gamma)}
\end{aligned}
$$

The supplier's profit in equilibrium is:

$$
\pi_s^F = \frac{(a-c)^2}{2(2+\gamma)}
$$

Firms' profits can be expressed as follows:

$$
\begin{aligned}
\pi_1^F &= \frac{(a-c)^2}{4(2+\gamma)^2} + F \\
\pi_2^F &= \frac{(a-c)^2}{4(2+\gamma)^2} + F
\end{aligned}
$$

At the first stage of the game, firm 2 accepts the licence if:

$$
F \leq \frac{(a-c)^2}{4(2+\gamma)^2}
$$

Substituting $F = \frac{(a-c)^2}{4(\gamma+2)^2}$ in profit equations, we obtain the following licensor's profit:

$$\pi_1^F = \frac{(a-c)^2}{2(2+\gamma)^2} \tag{37}$$

while $\pi_2^F = 0$.
Consumer surplus and social welfare are:

$$S^F = \frac{(a-c)^2(1+\gamma)}{4(2+\gamma)^2} \tag{38}$$

$$W^F = \frac{(a-c)^2(7+3\gamma)}{4(2+\gamma)^2} \tag{39}$$

In order to understand if for firm 1 it is profitable to license its technology to firm 2, we compare its monopolistic profit in (33) with that of licensing (37). We want also to understand if the licensor's choice is beneficial for social welfare, thus we analyze the following differentials that compare all voices of social welfare with those of monopoly.

$$\begin{aligned}
\pi_S^F - \pi_S^M &= \tfrac{(a-c)^2(2-\gamma)}{8(2+\gamma)} \geq 0 \\
\pi_1^F - \pi_1^M &= \tfrac{(a-c)^2(4-\gamma^2-4\gamma)}{16(2+\gamma)^2} \geq 0 \text{ if } \gamma < 0.828\,43 \\
S^F - S^M &= \tfrac{(a-c)^2(4-\gamma^2+4\gamma)}{32(2+\gamma)^2} \geq 0 \\
W^F - W^M &= \tfrac{(a-c)^2(28-7\gamma^2-4\gamma)}{32(2+\gamma)^2} \geq 0
\end{aligned}$$

We can state the following remark:

Remark 5 *If downstream firms face Cournot competition and the input market is monopolized by a supplier, firm 1 decides to licence its technology by means of a fixed fee if* $\gamma \leq 0.82843$. *Society is better off under licensing with respect to monopoly.*

Licensing with a per unit royalty By solving the maximization of profits in (31) and (32) with respect to quantities, we obtain the following reaction functions:

$$\begin{aligned}
q_1 &= \frac{a - \gamma q_2 - t_1}{2} \\
q_2 &= \frac{a - \gamma q_1 - t_2 - r}{2}
\end{aligned}$$

which exhibit strategic substitutability.

By solving the system of the previous reaction functions, we obtain the optimal quantities as functions of the royalty r and of the prices t_1 and t_2 set by the supplier:

$$\begin{aligned} q_1 &= \frac{a(2-\gamma)+\gamma(t_2+r)-2t_1}{4-\gamma^2} \\ q_2 &= \frac{a(2-\gamma)+\gamma t_1-2t_2-2r}{4-\gamma^2} \end{aligned}$$

The quantity produced by each firm decreases in its own input price and increases in the rival's input price. Also notice that a higher royalty rate positively (negatively) affects the licensor's (the licensee's) production.

At the previous stage of the game, firm S chooses the input prices, by maximizing its profit function in (27):

$$\begin{aligned} t_1 &= \frac{a+c}{2} \\ t_2 &= \frac{a+c-r}{2} \end{aligned}$$

The upstream monopolist applies two different input prices to downstream firms. The supplier charges to the licensor a price that is equal to that of monopoly while, being aware that the licensee has to pay a royalty to the patentee, she is induced to lower the rival's input price to keep up her demand.

By substituting these input prices in quantities functions, we obtain the following equilibrium quantities:

$$\begin{aligned} q_1 &= \frac{(a-c)(2-\gamma)+\gamma r}{2(4-\gamma^2)} \\ q_2 &= \frac{(a-c)(2-\gamma)-2r}{2(4-\gamma^2)} \end{aligned}$$

At the licensing stage, the maximization of firm 1's profit (31), after incorporating the optimal quantities, leads to the optimal royalty rate:

$$r^* = \frac{(a-c)(2-\gamma)(4-\gamma^2+\gamma)}{16-5\gamma^2}$$

Notice that $\frac{\partial r^*}{\partial \gamma} < 0$, namely firm 1 sets a royalty rate that is monotonically decreasing in γ. This implies that, when product substitutability increases, a lower royalty rate directly pushes towards an increase of the rival's production which raises licensing revenues. Moreover, by letting t_2 increase, a progressively lower royalty rate widens the firms' cost differences due to the payment of the input price and allows the patentee to keep an advantage on her own channel.

The equilibrium input prices are:

$$
\begin{aligned}
t_1^R &= \frac{a+c}{2} \\
t_2^R &= \frac{a(8-\gamma^3-2\gamma^2+2\gamma)+c(24+\gamma^3-8\gamma^2-2\gamma)}{2(16-5\gamma^2)}
\end{aligned}
$$

Analyzing the trend of the input price applied to the licensee, we observe that t_2^R increases in γ.

By substituting the optimal royalty r^* in the previous functions, we obtain the equilibrium quantities:

$$
\begin{aligned}
q_1^R &= \frac{(a-c)(8-\gamma^2-2\gamma)}{2(16-5\gamma^2)} \\
q_2^R &= \frac{(a-c)(4-3\gamma)}{2(16-5\gamma^2)}
\end{aligned}
$$

Firms' profits in equilibrium are:

$$
\pi_1^R = \frac{(a-c)^2(4-\gamma)(2-\gamma)}{4(16-5\gamma^2)} \tag{40}
$$

$$
\pi_2^R = \frac{(a-c)^2(4-3\gamma)^2}{4\left(16-5\gamma^2\right)^2} \tag{41}
$$

The supplier's profit is:

$$
\pi_s^R = \frac{(a-c)^2(80+4\gamma^4+6\gamma^3-35\gamma^2-24\gamma)}{2\left(16-5\gamma^2\right)^2} \tag{42}
$$

Consumer surplus and social welfare are:

$$
S^R = \frac{(a-c)^2(80+7\gamma^4+8\gamma^3-67\gamma^2+8\gamma)}{8\left(16-5\gamma^2\right)^2} \tag{43}
$$

$$
W^R = \frac{(a-c)^2(688+13\gamma^4+92\gamma^3-237\gamma^2-328\gamma)}{8\left(16-5\gamma^2\right)^2} \tag{44}
$$

Through a comparison of firm 1's profit and of all voices of social welfare between the case of licensing with a royalty and the monopoly case, we obtain the following differentials:

$$
\begin{aligned}
\pi_1^R-\pi_1^M &= \frac{(a-c)^2(16+9\gamma^2-24\gamma)}{16(16-5\gamma^2)} \geq 0 \\
\pi_S^R-\pi_S^M &= \frac{(a-c)^2(64-9\gamma^4+24\gamma^3+20\gamma^2-96\gamma)}{8(16-5\gamma^2)^2} \geq 0 \\
S^R-S^M &= \frac{(a-c)^2(64+3\gamma^4+32\gamma^3-108\gamma^2+32\gamma)}{32(16-5\gamma^2)^2} \geq 0 \\
W^R-W^M &= \frac{(a-c)^2(960-123\gamma^4+368\gamma^3+172\gamma^2-1312\gamma)}{32(16-5\gamma^2)^2} \geq 0
\end{aligned}
$$

and we can state the following remark:

Remark 6 *In the case of Cournot competition if the upstream market is monopolized by a supplier, royalty licensing is always advantageous for firm 1. This is a welfare improving choice with respect to no licensing regardless of the degree of product substitutability.*

Fixed fee vs. royalty Now we want to understand which is the optimal licensing contract for the patentee, assessing also welfare implications. We compare royalty and fixed fee licensing contracts and we obtain the following differentials:

$$\begin{aligned}
\pi_s^F - \pi_s^R &= \tfrac{(a-c)^2(96-4\gamma^5+11\gamma^4+23\gamma^3-66\gamma^2-32\gamma)}{2(2+\gamma)(16-5\gamma^2)^2} \geq 0 \\
\pi_1^F - \pi_1^R &= \tfrac{\gamma(a-c)^2(8+\gamma^3-2\gamma^2-2\gamma)}{4(2+\gamma)^2(16-5\gamma^2)} \leq 0 \\
S^F - S^R &= \tfrac{(a-c)^2(192-7\gamma^6+14\gamma^5+57\gamma^4-92\gamma^3-164\gamma^2+160\gamma)}{8(2+\gamma)^2(16-5\gamma^2)^2} \geq 0 \\
W^F - W^R &= \tfrac{(a-c)^2(832-13\gamma^6+6\gamma^5+167\gamma^4-52\gamma^3-668\gamma^2+96\gamma)}{8(2+\gamma)^2(16-5\gamma^2)^2} \geq 0
\end{aligned}$$

In order to summarize the previous results, we state the following proposition:

Proposition 3 *In the case of input market monopoly, if downstream firms compete with respect to quantities in the interval* $\gamma \leq 0.82843$ *a royalty is preferred to a fixed fee contract by the patentee. Society is better off under a fixed payment.*

Proof. It follows from a) a comparison between the patentee's profits in (37) with those in (42); b) a comparison of social welfare in (39) with that in (44).

2.2.3 Licensing under Bertrand competition

We assume that at the downstream competition stage firms play *à la* Bertrand facing the direct demand functions (1) and (2).

Licensing through a fixed fee At the last stage of the game, firms maximize their profits in (29) and (30) with respect to prices. We obtain the following reaction functions:

$$\begin{aligned}
p_1 &= \frac{a(1-\gamma) + \gamma p_2 + t_1}{2} \\
p_2 &= \frac{a(1-\gamma) + \gamma p_1 + t_2}{2}
\end{aligned}$$

which exhibit strategic complementarity. By solving the system of the two reaction functions, we obtain the optimal prices as functions of the input prices:

$$\begin{aligned} p_1 &= \frac{a(2+\gamma)(1-\gamma)+\gamma t_2+2t_1}{4-\gamma^2} \\ p_2 &= \frac{a(2+\gamma)(1-\gamma)+\gamma t_1+2t_2}{4-\gamma^2} \end{aligned}$$

Notice that each firm's final price increases in both upstream wholesale prices.

At the second stage, by incorporating the optimal prices the supplier S maximizes her profit in (27) with respect to the input prices. The equilibrium prices of the intermediate product set by the supplier are:

$$\begin{aligned} t_1^F &= \frac{a+c}{2} \\ t_2^F &= \frac{a+c}{2} \end{aligned}$$

By substituting input prices in the previous equations of prices, we obtain:

$$\begin{aligned} p_1^F &= \frac{a(3-2\gamma)+c}{2(2-\gamma)} \\ p_R^F &= \frac{a(3-2\gamma)+c}{2(2-\gamma)} \end{aligned}$$

The supplier's profit in equilibrium is:

$$\pi_s^F = \frac{(a-c)^2}{2\,(1+\gamma)\,(2-\gamma)}$$

Downstream firms' profits are:

$$\begin{aligned} \pi_1^F &= \frac{(a-c)^2(1-\gamma)}{4\,(2-\gamma)^2\,(1+\gamma)} + F \\ \pi_2^F &= \frac{(a-c)^2(1-\gamma)}{4\,(2-\gamma)^2\,(1+\gamma)} - F \end{aligned}$$

The maximum level of F that firm 2 pays is the amount that makes it indifferent between licensing and no licensing:

$$F = \frac{(a-c)^2(1-\gamma)}{4\,(2-\gamma)^2\,(1+\gamma)}$$

We obtain the following firms' profits:

$$\pi_1^F = \frac{(1-\gamma)(a-c)^2}{2\left(2-\gamma\right)^2\left(1+\gamma\right)} \tag{45}$$

where $\pi_2^F = 0$.
Consumer surplus and social welfare can be written:

$$S^F = \frac{\left(a-c\right)^2}{4\left(2-\gamma\right)^2\left(1+\gamma\right)} \tag{46}$$

$$W^F = \frac{(7-4\gamma)(a-c)^2}{4\left(2-\gamma\right)^2\left(1+\gamma\right)} \tag{47}$$

By comparing this setting with that of monopoly, we obtain the following differentials:

$$\begin{aligned}
\pi_S^F - \pi_S^M &= \tfrac{(a-c)^2(2+\gamma^2-\gamma)}{8(1+\gamma)(2-\gamma)} \geq 0 \\
\pi_1^F - \pi_1^M &= \tfrac{(a-c)^2(4-\gamma^3+3\gamma^2-8\gamma)}{16(2-\gamma)^2(1+\gamma)} \geq 0 \text{ if } \gamma < 0.611\,71 \\
S^F - S^M &= \tfrac{(a-c)^2(4-\gamma^3+3\gamma^2)}{32(2-\gamma)^2(1+\gamma)} \geq 0 \\
W^F - W^M &= \tfrac{(a-c)^2(28-7\gamma^3+21\gamma^2-32\gamma)}{32(2-\gamma)^2(1+\gamma)} \geq 0
\end{aligned}$$

and we can assert the following remark:

Remark 7 *If firms engage in price competition and the input market is monopolized by a supplier, firm 1 licences its technology with a fixed fee if* $\gamma \leq 0.61171$. *This is a welfare improving strategy with respect to no licensing.*

Consistently with the case of upstream market competition, but in contrast to Arya and Mittendorf (2006)'s result, we observe that if products are sufficiently differentiated, for the innovator it is profitable to license with fixed fee, regardless of the mode of competition.[8] Thus, their result that the patentee prefers not to license by means of a fixed payment is conditioned on their assumption of homogeneous products.

Licensing with a per unit royalty At the market competition stage, firms maximize their profits in (31) and (32) with respect to prices. The price reaction functions are:

$$\begin{aligned}
p_1 &= \frac{a(1-\gamma)+\gamma p_2+t_1+r\gamma}{2} \\
p_2 &= \frac{a(1-\gamma)+\gamma p_1+t_2+r}{2}
\end{aligned}$$

[8] We observe the same γ limits obtained in the case of perfectly competitive input market: for downstream cost symmetry, the monopolist sets to both firms the same input price, equal to that applied when firm 1 is monopolist.

which exhibit strategic complementarity.

By solving the system of the two reaction functions, we obtain the equilibrium prices:

$$
\begin{aligned}
p_1 &= \frac{a(2+\gamma)(1-\gamma)+\gamma t_2+2t_1+3r\gamma}{4-\gamma^2} \\
p_2 &= \frac{a(2+\gamma)(1-\gamma)+\gamma t_1+2t_2+r(2+\gamma^2)}{4-\gamma^2}
\end{aligned}
$$

Clearly, each firm's final price increases in both upstream wholesale prices and, moreover, in the royalty rate.

By solving the maximization of profits in (27), we find the optimal discriminatory prices set by the monopolist supplier, which are as follows:

$$
\begin{aligned}
t_1 &= \frac{a+c-r\gamma}{2} \\
t_2 &= \frac{a+c-r}{2}
\end{aligned}
$$

Notice that both firms' input prices decrease if the royalty imposed by the licensor increases. It is easy to notice that t_1^R depends also on the degree of product substitutability, which entails that the wholesale price charged to the licensor is lower, the higher is product substitutability.

By substituting these input prices in downstream prices functions, we obtain the following equilibrium prices:

$$
\begin{aligned}
p_1 &= \frac{a(6-2\gamma^2-\gamma)+c(2+\gamma)+3\gamma r}{2(4-\gamma^2)} \\
p_2 &= \frac{a(6-2\gamma^2-\gamma)+c(2+\gamma)+r(2+\gamma^2)}{2(4-\gamma^2)}
\end{aligned}
$$

At the licensing game, maximization of firm 1's profit in (31), incorporating optimal prices, gives the equilibrium royalty rate:

$$
r^* = \frac{(a-c)(2+\gamma)(4-\gamma)}{(16-\gamma^4+3\gamma^2)}
$$

As shown in Figure 2, we prove that, as long as products are sufficiently differentiated, the patentee sets a royalty rate that increases in γ, which contrasts with the quantity competition case. Indeed, given complementarity of prices under Bertrand, the patentee can keep her profits on both channels relatively high by raising the royalty rate and letting t_1 and t_2 decrease, which also mitigates the double marginalization problem. However, when products become less differentiated, a decreasing royalty rate is aimed to soften harsher downstream competition through higher input prices.

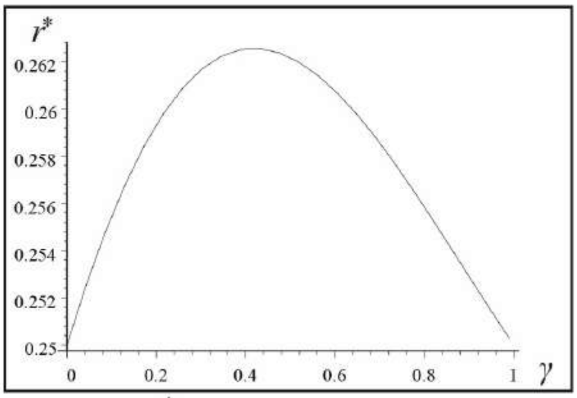

Fig. 2: The trend of r^* in case of upstream monopolist that applies price discrimination under Bertrand competition.

The equilibrium input prices are:

$$
\begin{aligned}
t_1^R &= \tfrac{a(16-\gamma^4+\gamma^3+\gamma^2-8\gamma)+c(16-\gamma^4-\gamma^3+5\gamma^2+8\gamma)}{2(16-\gamma^4+3\gamma^2)} \\
t_2^R &= \tfrac{a(8-\gamma^4+4\gamma^2-2\gamma)+c(24-\gamma^4+2\gamma^2+2\gamma)}{2(16-\gamma^4+3\gamma^2)}
\end{aligned}
$$

It is worth observing that while t_1^R decreases in γ, t_2^R is not monotone: as long as products are sufficiently differentiated t_2^R decreases (for increasing values of r), while when γ assumes higher values, the licensee's input price increases (for decreasing values of r).

By substituting this value of r^* in prices functions, we obtain the following equilibrium variables:

$$
\begin{aligned}
p_1^R &= \frac{a(24-2\gamma^4-\gamma^3+4\gamma^2+2\gamma)+c(8+\gamma^3+2\gamma^2-2\gamma)}{2(16-\gamma^4+3\gamma^2)} \\
p_2^R &= \frac{a(28-2\gamma^4-\gamma^3+5\gamma^2-3\gamma)+c(4+\gamma^3+\gamma^2+3\gamma)}{2(16-\gamma^4+3\gamma^2)}
\end{aligned}
$$

Downstream firms' profits are:

$$
\pi_1^R = \frac{(a-c)^2(8+\gamma^3+3\gamma^2+6\gamma)}{4\,(1+\gamma)\,(16-\gamma^4+3\gamma^2)} \tag{48}
$$

$$
\pi_2^R = \frac{(a-c)^2(4-\gamma^4-\gamma^3+3\gamma^2-5\gamma)(4+\gamma^3+2\gamma^2-\gamma)}{4\,(1+\gamma)\,(16-\gamma^4+3\gamma^2)^2} \tag{49}
$$

In the same way we calculate the supplier's profit:

$$
\pi_s^R = \frac{(a-c)^2(160-2\gamma^7-2\gamma^6+5\gamma^5-11\gamma^4+18\gamma^3+42\gamma^2-48\gamma)}{4\,(1+\gamma)\,(16-\gamma^4+3\gamma^2)^2} \tag{50}
$$

Consumer surplus and social welfare are:

$$S^R = \frac{(a-c)^2(80+2\gamma^6+6\gamma^5+6\gamma^4+31\gamma^3+29\gamma^2+8\gamma)}{8(1+\gamma)(16-\gamma^4+3\gamma^2)^2} \tag{51}$$

$$W^R = \frac{(a-c)^2(688-8\gamma^7-14\gamma^6+14\gamma^5-18\gamma^4+109\gamma^3+307\gamma^2+56\gamma)}{8(1+\gamma)(16-\gamma^4+3\gamma^2)^2} \tag{52}$$

Through a comparison of firm 1's profit in case of royalty licensing in (48) and in case of no licensing (33) and comparing all voices of social welfare with those observed in absence of licensing, we get the following differentials:

$$\begin{aligned}
\pi_1^R - \pi_1^M &= \frac{(a-c)^2(16+\gamma^3+9\gamma^2+8\gamma+\gamma^4+\gamma^5)}{16(1+\gamma)(16-\gamma^4+3\gamma^2)} \geq 0 \\
\pi_S^R - \pi_S^M &= \frac{(a-c)^2(64-\gamma\sigma)}{8(1+\gamma)(16-\gamma^4+3\gamma^2)^2} \geq 0 \text{ if } \gamma \leq 0.179\,75 \\
S^R - S^M &= \frac{(a-c)^2(64-\gamma\varpi)}{32(1+\gamma)(16-\gamma^4+3\gamma^2)^2} \geq 0 \text{ if } \gamma \leq 0.299\,3 \\
W^R - W^M &= \frac{(a-c)^2(960-\gamma\mu)}{32(1+\gamma)(16-\gamma^4+3\gamma^2)^2} \geq \text{if } \leq 0.928\,18
\end{aligned}$$

where
$\sigma = (352 + \gamma^8 + \gamma^7 - 2\gamma^6 - 2\gamma^5 - 33\gamma^4 - \gamma^3 + 60\gamma^2 + 12\gamma)$
$\varpi = (224 + \gamma^8 + \gamma^7 - 6\gamma^6 - 14\gamma^5 - 47\gamma^4 - 47\gamma^3 - 28\gamma^2 - 20\gamma)$
$\mu = (1568 + 7\gamma^8 + 7\gamma^7 - 10\gamma^6 + 14\gamma^5 - 217\gamma^4 - 89\gamma^3 + 236\gamma^2 - 556\gamma)$

We can now state the following remark:

Remark 8 *In the case of price competition if the input market is monopolized by a supplier, licensing with a per unit royalty is profitable for firm 1 regardless of the product differentiation parameter. This is a welfare improving strategy as long as $\gamma \leq 0.927$, otherwise this choice leads society to be worse off with respect to no licensing.*

We have observed that from the perspective of the patentee licensing with a royalty is always profitable regardless of the type of market competition, but if firms compete in prices it can be a welfare detrimental strategy with respect to no licensing.[9] The choice of the optimal royalty rate allows the patentee to influence the supplier's input prices and is aimed to induce sufficiently high licensing revenues by pushing towards a relevant licensee's output (without limiting the licensor's production) in Cournot and sufficiently high downstream prices in Bertrand. This results in a welfare-enhancing market outcome with respect to no licensing in Cournot. In Bertrand, by contrast, it causes a welfare-detrimental outcome when products are close substitutes and the aim to soften retail market competition is achieved by letting the input prices be relatively high, thus negatively impacting on social welfare through an increase of double

[9] If the analysis is carried out introducing a fixed amount F, with an upstream monopolist that applies discriminatory prices, a two-part tariff is always advantageous for the innovator and it is a welfare improving strategy regardless of the mode of competition.

marginalization. In this regard, it is worth noting that our result on welfare-reducing royalty licensing under Bertrand resembles that found by Faulí-Oller and Sandonis (2002) in a two-part licensing framework with process innovation. With respect to the latter, which focuses on the role played by the size of process innovation in delivering the main result, our model highlights the role of vertical relations in causing licensing to be welfare-detrimental in a product innovation framework.[10] Notice that the welfare-detrimental effect of licensing under royalty is not observed when the upstream monopolist sets a non-discriminatory input price to both retailers, as shown in the Appendix A. In this case, indeed, the patentee softens downstream competition by raising the royalty rate regardless of the degree of product substitutability. This contributes to reduce the upstream input price and, consequently, double marginalization even when products are close substitutes.

Fixed fee vs. royalty. By comparing firm 1's profit in case of fixed fee (45) with that obtained with royalty licensing in (48) and all voices of social welfare in the two different licensing contracts, we obtain the following differentials:

$$
\begin{aligned}
\pi_s^F - \pi_s^R &= \frac{(a-c)^2\left(192+2\gamma^6-5\gamma^5-16\gamma^4+10\gamma^3-4\gamma^2+64\gamma\right)}{4(16-\gamma^4+3\gamma^2)^2(2-\gamma)} \geq 0 \\
\pi_1^F - \pi_1^R &= -\frac{\gamma(a-c)^2(24-\gamma^4+\gamma^3+4\gamma^2-10\gamma)}{4(2-\gamma)^2(1+\gamma)(16-\gamma^4+3\gamma^2)} \leq 0 \\
S^F - S^R &= \frac{(a-c)^2\left(192+2\gamma^6-4\gamma^5-27\gamma^4+52\gamma^3-68\gamma^2+96\gamma\right)}{8(16-\gamma^4+3\gamma^2)^2(2-\gamma)^2} \geq 0 \\
W^F - W^R &= \frac{(a-c)^2\left(832-4\gamma^7+14\gamma^6+32\gamma^5-85\gamma^4-36\gamma^3+4\gamma^2-352\gamma\right)}{8(16-\gamma^4+3\gamma^2)^2(2-\gamma)^2} \geq 0
\end{aligned}
$$

We summarize the above results in the following proposition:

Proposition 4 *In the case of downstream price competition with an input market monopolized by a supplier in the interval* $\gamma \leq 0.61171$ *royalty licensing is preferred to fixed fee licensing regardless of the product differentiation parameter. Society is better off under a fixed fee contract.*

Proof. It follows from a) a comparison between the patentee's profits in (45) with those in (48); b) a comparison of social welfare in (47) with that in (52).

Comparing the two types of licensing contracts, we observe that the patentee prefers a per unit royalty to a fixed payment, regardless of the degree of product substitutability and the mode of market competition. Indeed, by setting a

[10] We have also performed the analysis in Section 2.2 on the assumption of licensing a non-drastic process innovation. This amounts to introducing a vertical supply relationship in the Faulí-Oller and Sandonis (2002)'s framework under a zero fixed-fee. In such a framework, we still observe a welfare-detrimental licensing result in Bertrand, which is more likely (indeed, it occurs in a wider range of the product differentiation parameter) than in either Section 2.2 or Faulí-Oller and Sandonis (2002), due to the negative impact of a reduced 'competition effect' and double marginalization on social welfare.

variable royalty, the patentee can influence the supplier's input prices, thus allowing the licensee (under quantity competition) or both firms (under price competition) to benefit from lower input costs and orienting downstream firms' strategic decisions towards the most profitable outcome.

3 An extension: the incumbent licenses one of two varieties

In this section we sketch the results obtained when we assume the patentee produces two varieties prior to licensing one of them to a potential rival. Under such an assumption, the 'variety effect', which has been shown in the above sections to cause profitability of licensing when it overcomes the 'competition effect', cancels out. This makes licensing through a fixed fee never optimal in this setting, regardless of the mode of competition and the upstream market structure. By contrast, when royalty licensing is at stake, it turns out to be profitable in both Cournot and Bertrand, provided that the upstream market is a monopoly and the positive 'vertical efficiency' effect overcomes the negative competition effect. By assessing the welfare implications of licensing in a scenario with two *ex-ante* varieties, we observe that the mode of competition matters in letting royalty licensing affect social welfare. Indeed, the absence of a variety effect dampens the welfare-enhancing character of licensing, leading on the one hand the positive effect of competition in Cournot to dominate the negative effect of double marginalization only under sufficiently low product differentiation (i.e., $\gamma > 0.71$).[11] On the other hand, it causes the negative effect of double marginalization in Bertrand to always dominate the competition effect, which is further limited by the incentive to keep the market prices higher through royalty licensing.[12] This result suggests that, under royalty licensing, a higher degree of product substitutability is more likely to improve social welfare in Cournot and to hurt social welfare in Bertrand.[13] See the Appendix B for details.

4 Concluding remarks

In this work we have studied the optimal licensing behavior of a product innovator which competes either *à la* Cournot or *à la* Bertrand against a licensee

[11] By contrast, by assuming that the incumbent produces one variety only prior to licensing, we obtained that the positive competition effect dominates the negative double marginalization effect for any degree of product differentiation.

[12] Under one *ex-ante* variety, we conversely found that the negative effect of double marginalization dominates the positive competition effect when the degree of product differentiation is very high.

[13] By assuming a two part tariff contract, we observe that licensing is advantageous for the patentee in the presence of an upstream monopoly only. If firms compete à la Cournot, licensing leads society to be worse off when the degree of product differentiation is sufficiently high. Under Bertrand competition, conversely, a two-part tariff is always a welfare-detrimental strategy.

in a vertical market with a monopoly supplier. By dealing with both Cournot and Bertrand downstream competition under imperfect substitutes, our model has extended the quantity competition model of Arya and Mittendorf (2006) to both price competition and differentiated products. We have highlighted the optimality of fixed fee licensing when product differentiation is sufficiently high, which enables the innovator to keep her licensing returns relatively high without dampening significantly her own market profit under firm cost symmetry. This result, which holds regardless of the mode of competition, proves that the Arya and Mittendorf's finding that a fixed fee is never profitable is conditioned on their assumption of homogeneous products. However, both product differentiation and the mode of competition do not play any role in letting a fixed fee contract be always dominated by a royalty contract. The latter, indeed, allows the patentee to strategically orient to her own benefit the input price's choice by the upstream supplier, thus worsening the social welfare conditions with respect to a fixed fee. By contrast, in the case of perfectly competitive upstream market the profit dominance of a royalty over a fixed fee contract appears only for a sufficiently high degree of the product differentiation parameter. Finally, by assessing the welfare implications of licensing, we have pointed out the circumstances under which royalty licensing hurts social welfare with respect to no licensing. This occurs when firms compete in prices and product substitutability is very high, which pushes the licensor to charge a relatively low royalty rate and soften downstream competition through higher input prices. By worsening the double marginalization problem, royalty licensing turns out to be welfare-detrimental as compared to the monopoly case. The result is in contrast with both Arya and Mittendorf (2006), which focus on licensing as a welfare-improving strategy, and the case of a perfectly competitive upstream market.

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Appendix A

Following the standard procedure described above, we analyze the game developed in the Subsection 2.2.3 under royalty licensing, assuming that the upstream monopolist does not set discriminatory prices. Firm 1 receives a royalty r for each unit of the quantity produced by firm 2 and, to produce the final products, they buy the input from the supplier with a price t.

Firms' profits are:

$$\begin{aligned}\pi_1 &= (p_1 - t)q_1 + rq_2 && \text{(A1)}\\ \pi_2 &= (p_2 - t)q_2 - rq_2 && \text{(A2)}\end{aligned}$$

The profit of the input supplier S can be written:

$$\pi_S = (t - c)\, q_1 + (t - c)\, q_2 \tag{A3}$$

By solving the maximization of (A1) and (A2) with respect to p_1 and p_2, we obtain the following reaction functions:

$$\begin{aligned}p_1 &= \frac{a(1-\gamma) + \gamma p_2 + t + r\gamma}{2}\\ p_2 &= \frac{a(1-\gamma) + \gamma p_1 + t + r}{2}\end{aligned}$$

which exhibit strategic complementarity. By solving the system of the two reaction functions, we obtain the equilibrium prices:

$$\begin{aligned}p_1 &= \frac{a(2-\gamma-\gamma^2) + t(2+\gamma) + 3r\gamma}{4-\gamma^2}\\ p_2 &= \frac{a(2-\gamma-\gamma^2) + t(2+\gamma) + r(2+\gamma^2)}{4-\gamma^2}\end{aligned}$$

Notice that each firm's final price increases in both the input price and the royalty rate.

By maximizing her profit, the input supplier sets the following price:

$$t = \frac{2(a+c) - r(1+\gamma)}{4} \tag{A4}$$

Notice that t decreases in r and in γ.

By substituting this input price in the price functions, we obtain the following equilibrium prices:

$$
\begin{aligned}
p_1 &= \frac{2a(6-2\gamma^2-\gamma)+2c(2+\gamma)-r(2+\gamma^2-9\gamma)}{4(4-\gamma^2)} \\
p_2 &= \frac{2a(6-2\gamma^2-\gamma)+2c(2+\gamma)+3r(2+\gamma^2-\gamma)}{4(4-\gamma^2)}
\end{aligned}
$$

Given (A4), firm 1 chooses the value of r that maximizes its total income (A3), obtaining the optimal royalty:

$$
r^* = \frac{2(a-c)(2+\gamma)(10+\gamma^2-5\gamma)}{92-3\gamma^4+19\gamma^2-36\gamma} \tag{A5}
$$

As shown in Figure A1, the patentee sets a royalty rate that generally increases in γ, while it decreases in a small interval of the values of γ, that is when products are very close substitutes.

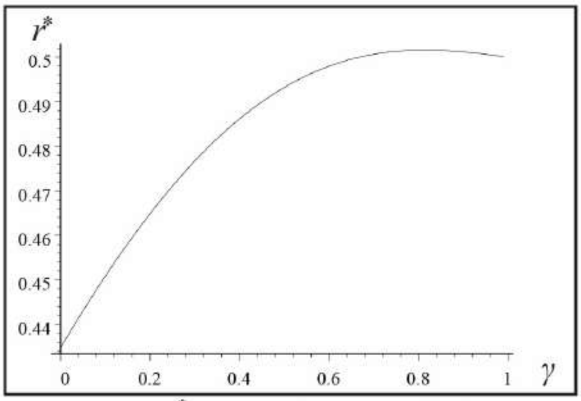

Fig. A1: The trend of r^* in case of upstream monopolist that doesn't apply price discrimination under Bertrand competition.

We can write the equilibrium input price as follows:

$$
t = \frac{a(36-2\gamma^4+\gamma^3+11\gamma^2-28\gamma)+c(56-\gamma^4-\gamma^3+8\gamma^2-8\gamma)}{(92-3\gamma^4+19\gamma^2-36\gamma)}
$$

It is easy to observe that the input price decreases in γ.

By substituting this value of r^* in the price functions, we obtain the following market variables:

$$
\begin{aligned}
p_1^R &= \frac{a(64-3\gamma^4-\gamma^3+10\gamma^2-16\gamma)+c(28+\gamma^3+9\gamma^2-20\gamma)}{92-3\gamma^4+19\gamma^2-36\gamma} \\
p_2^R &= \frac{a(84-3\gamma^4-3\gamma^3+22\gamma^2-46\gamma)+c(8+3\gamma^3-3\gamma^2+10\gamma)}{92-3\gamma^4+19\gamma^2-36\gamma}
\end{aligned}
$$

We obtain the following profits:

$$\pi_1^R = \frac{(a-c)^2(12+\gamma^3+\gamma^2+4\gamma)}{(1+\gamma)(92-3\gamma^4+19\gamma^2-36\gamma)} \tag{A6}$$

$$\pi_2^R = \frac{(a-c)^2(8-\gamma^4-6\gamma^3+17\gamma^2-18\gamma)(8+\gamma^3+7\gamma^2-10\gamma)}{(1+\gamma)(92-3\gamma^4+19\gamma^2-36\gamma)^2} \tag{A7}$$

$$\pi_s^R = \frac{2(a-c)^2(36-2\gamma^4+\gamma^3+11\gamma^2-28\gamma)(18+2\gamma^3+3\gamma^2-5\gamma)}{(1+\gamma)(92-3\gamma^4+19\gamma^2-36\gamma)^2} \tag{A8}$$

Consumer surplus and social welfare are:

$$S^R = \frac{(a-c)^2(424+3\gamma^6+22\gamma^5-55\gamma^4+132\gamma^3+78\gamma^2-280\gamma)}{(1+\gamma)(92-3\gamma^4+19\gamma^2-36\gamma)^2} \tag{A9}$$

$$W^R = \frac{(a-c)^2(2888-12\gamma^7-21\gamma^6+84\gamma^5-83\gamma^4-170\gamma^3+1518\gamma^2-1936\gamma)}{(1+\gamma)(92-3\gamma^4+19\gamma^2-36\gamma)^2} \tag{A10}$$

By comparing firm 1's profit (A6) with (33) and all voices of social welfare with those of no licensing, we obtain the following differentials:

$$\begin{aligned}
\pi_S^R - \pi_S^M &= \frac{(a-c)^2(1904-\gamma\delta)}{8(1+\gamma)(92-3\gamma^4+19\gamma^2-36\gamma)^2} \geq 0 \text{ if } \gamma \leq 0.167\,01 \\
\pi_1^R - \pi_1^M &= \frac{(a-c)^2(100+3\gamma^5+3\gamma^4-3\gamma^3+33\gamma^2+8\gamma)}{16(1+\gamma)(92-3\gamma^4+19\gamma^2-36\gamma)} \geq 0 \\
S^R - S^M &= \frac{(a-c)^2(5104-\gamma\rho)}{32(1+\gamma)(92-3\gamma^4+19\gamma^2-36\gamma)^2} \geq 0 \text{ if } \gamma \leq 0.704\,39 \\
W^R - W^M &= \frac{(a-c)^2(33\,168-\gamma\theta)}{32(1+\gamma)(92-3\gamma^4+19\gamma^2-36\gamma)^2} \geq 0
\end{aligned}$$

where

$\delta = (12784 + 9\gamma^8 + 9\gamma^7 - 50\gamma^6 + 166\gamma^5 - 535\gamma^4 - 535\gamma^3 + 4208\gamma^2 - 8968\gamma)$
$\rho = (10800 + 9\gamma^8 + 9\gamma^7 - 114\gamma^6 + 6\gamma^5 - 679\gamma^4 + 201\gamma^3 - 800\gamma^2 - 4328\gamma)$
$\theta = (74832 + 63\gamma^8 + 63\gamma^7 - 414\gamma^6 + 1386\gamma^5 - 2513\gamma^4 - 8257\gamma^3 + 29408\gamma^2 - 61400\gamma)$

We can state the following remark:

Remark A1 *If the input market is monopolized by a supplier that applies the same price to both downstream firms, licensing with royalty is always profitable. Royalty licensing leads society to be better off.*

Appendix B

In this Appendix we assume that the patentee licenses one of her two varieties, analyzing both the case of perfect competition and monopoly in the usptream market.

Upstream perfectly competitive market

In the case of no licensing the firm's 1 profit is:

$$\pi_1 = (p_1 - c)\, q_1 + (p_2 - c)\, q_2 \tag{B1}$$

Through the standard maximization we obtain the firm 1's profit as follows:

$$\pi_1 = \frac{(a-c)^2}{2(1+\gamma)} \tag{B2}$$

Consumer surplus and social welfare are:

$$\begin{aligned} S &= \frac{(a-c)^2}{4(1+\gamma)} \\ W &= \frac{3\,(a-c)^2}{4(1+\gamma)} \end{aligned}$$

Licensing under Cournot competition

In this subgame we assume that firms play *à la* Cournot. By comparing the firm 1's profit in (B2) with the profit gains in (13) and (16) under fixed fee and royalty licensing, respectively, we obtain the following differentials:

$$\begin{aligned} \pi_1^F - \pi_1^M &= -\tfrac{\gamma^2 (a-c)^2}{2(1+\gamma)(\gamma+2)^2} < 0 \\ \pi_1^R - \pi_1^M &= -\tfrac{(a-c)^2 (4-\gamma^3+\gamma^2-4\gamma)}{4(1+\gamma)(8-3\gamma^2)} < 0 \end{aligned}$$

and we state the following remark:

Remark B1 *If firms engage in quantity competition, fixed fee and royalty licensing are never profitable for the patentee with respect to no licensing.*

Licensing under Bertrand competition

In this subgame we assume that firms face price competition. By comparing the firm 1's profit in (B2) with the profit gains in (20) and (23) under licensing by means of a fixed fee and a royalty, respectively, we have the following differentials:

$$\begin{aligned} \pi_1^F - \pi_1^M &= -\tfrac{\gamma^2 (a-c)^2}{2(1+\gamma)(2-\gamma)^2} < 0 \\ \pi_1^R - \pi_1^M &= -\tfrac{(a-c)^2 (4-\gamma^3+\gamma^2-4\gamma)}{4(1+\gamma)(8+\gamma^2)} < 0 \end{aligned}$$

and we state the following remark:

Remark B2 *If firms compete à la Bertrand, licensing is never profitable for the patentee with respect to no licensing, both by means of a royalty and a fixed fee.*

The upstream monopoly case

The firm's 1 profit can be written as follows:

$$\pi_1 = (p_1 - t)q_1 + (p_2 - t)q_2 \tag{B3}$$

and the upstream supplier's profit is:

$$\pi_S = (t - c)(q_1 + q_2) \tag{B4}$$

Through the standard maximization we obtain the following firms' profits as follows:

$$\pi_1^M = \frac{(a-c)^2}{8(1+\gamma)} \tag{B5}$$

$$\pi_S^M = \frac{(a-c)^2}{4(1+\gamma)} \tag{B6}$$

Consumer surplus and social welfare are:

$$S^M = \frac{(a-c)^2}{16(1+\gamma)} \tag{B7}$$

$$W^M = \frac{7(a-c)^2}{16(1+\gamma)} \tag{B8}$$

Licensing under Cournot competition

In this subgame we assume that firms play *à la* Cournot. In order to understand if for firm 1 it is profitable to license its technology to firm 2 with a fixed fee, we compare its monopolistic profit in (B5) with that of licensing (37).

$$\pi_1^F - \pi_1^M = -\frac{\gamma^2(a-c)^2}{8(1+\gamma)(2+\gamma)^2} < 0$$

In order to understand if licensing with a royalty is advantageous for the innovator we compare firm 1's profit in (B5) with that in (40). We compare also the voices of social welfare in the case of royalty licensing with those under monopoly and we obtain the following differentials:

$$\begin{aligned}
\pi_1^R - \pi_1^M &= \frac{\gamma(a-c)^2(4+2\gamma^2-5\gamma)}{8(1+\gamma)(16-5\gamma^2)} > 0 \\
\pi_S^R - \pi_S^M &= -\frac{(a-c)^2(96-8\gamma^5+5\gamma^4+58\gamma^3-42\gamma^2-112\gamma)}{4(16-5\gamma^2)^2(1+\gamma)} > 0 \text{ if } \gamma > 0.936\,45 \\
S^R - S^M &= -\frac{(a-c)^2(96-14\gamma^5-5\gamma^4+118\gamma^3-42\gamma^2-176\gamma)}{16(16-5\gamma^2)^2(1+\gamma)} > 0 \text{ if } \gamma > 0.591\,46 \\
W^R - W^M &= -\frac{(a-c)^2(416-26\gamma^5-35\gamma^4+290\gamma^3+10\gamma^2-720\gamma)}{16(1+\gamma)(16-5\gamma^2)^2} \text{ if } \gamma > 0.710\,14
\end{aligned}$$

and we state the following remark:

Remark B3 *If firms compete à la Cournot, licensing with a fixed fee is never profitable for the innovator, while royalty licensing is always advantageous with respect to no licensing. Royalty licensing improves social welfare with respect to no licensing only if* $\gamma > 0.71$.

Licensing under Bertrand competition

In this subgame we assume that firms face price competition. By comparing the firm 1's profit in (B5) with that under fixed fee licensing in (45) we obtain the following differential:

$$\pi_1^F - \pi_1^M = -\frac{\gamma^2(a-c)^2}{8(1+\gamma)(2-\gamma)^2} < 0$$

By comparing firm 1's profit and of all components of social welfare between the case of licensing with a royalty and the monopoly case, we obtain the following differentials:

$$\begin{aligned}
\pi_1^R - \pi_1^M &= \frac{\gamma(a-c)^2(12+\gamma^3+2\gamma^2+3\gamma)}{8(1+\gamma)(16-\gamma^4+3\gamma^2)} > 0 \\
\pi_S^R - \pi_S^M &= -\frac{(a-c)^2(96+\gamma^8+2\gamma^7-4\gamma^6-5\gamma^5-12\gamma^4-18\gamma^3+54\gamma^2+48\gamma)}{4(1+\gamma)(16-\gamma^4+3\gamma^2)^2} < 0 \\
S^R - S^M &= -\frac{(a-c)^2(96+\gamma^8-10\gamma^6-12\gamma^5-35\gamma^4-62\gamma^3+38\gamma^2-16\gamma)}{16(1+\gamma)(16-\gamma^4+3\gamma^2)^2} < 0 \\
W^R - W^M &= -\frac{(a-c)^2(416+7\gamma^8+16\gamma^7-14\gamma^6-28\gamma^5-125\gamma^4-218\gamma^3+58\gamma^2-112\gamma)}{16(1+\gamma)(16-\gamma^4+3\gamma^2)^2} < 0
\end{aligned}$$

and we state the following remark:

Remark B4 *If firms compete à la Bertrand, licensing by means of a fixed fee is never advantageous, while royalty licensing is always profitable for the patentee. Under licensing by means of a royalty, society is worse off with respect to no licensing.*